# *Introduction into design engineering week 9*

Dr. Yukari AOKI

# ***Stress on an oblique plane under axial loading***

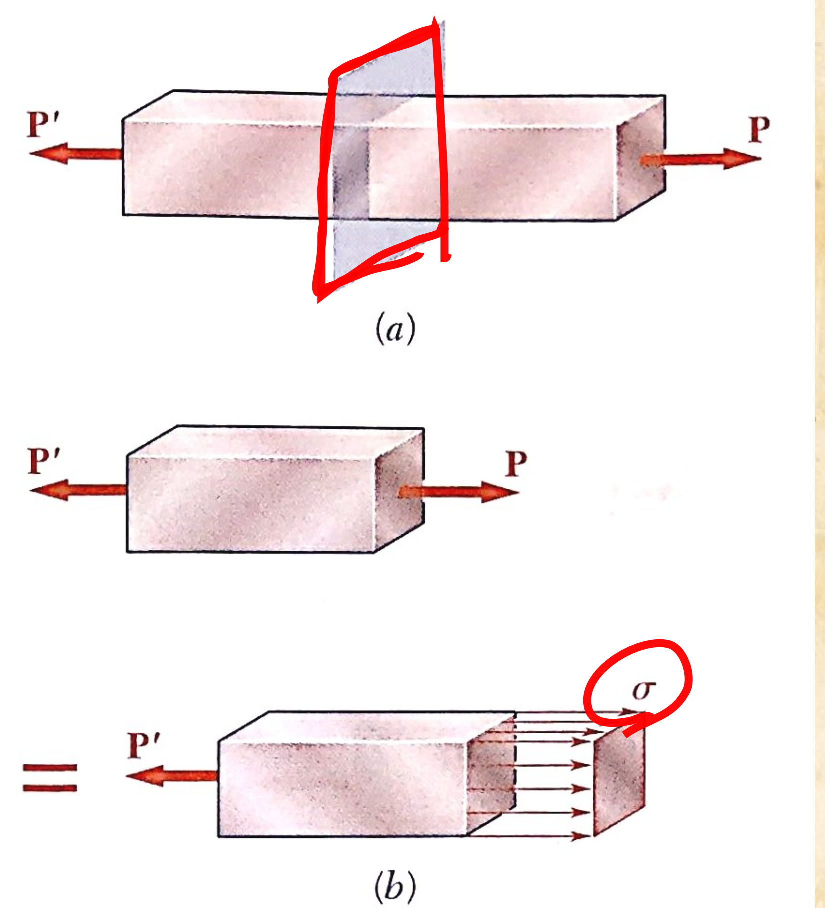


Fig. 1.28

- In the previous sessions, **axial force** exerted on a *two-force member* (see in Fig.1.28a) were found to cause **normal stress** in that member (in Fig.1.28b).

- While **transverse forces** exerted on *bolts and pins* (Fig.1.29a) were found to cause **shearing stresses** in those connections (Fig.1.29b).

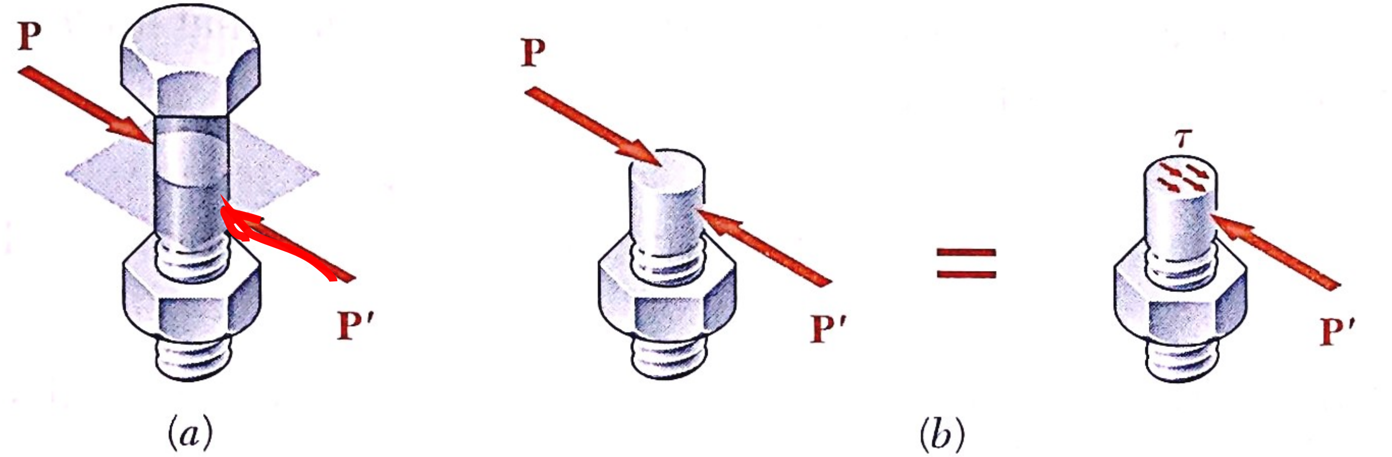


Fig. 1.29

- The reason such a relation was observed between **axial forces** and **normal stresses** on one hand, and **transverse forces** and s**hearing stresses** on the other, was because stresses were being determined only on planes perpendicular to the axis of the member or connection.
- As it will be seen in this section, ***axial forces*** cause both ***normal and shearing stresses*** on planes which are not perpendicular to the axis of the member.

- Similarly, ***transverse forces*** exerted on **a bolt or a pin** cause both ***normal and shearing stresses*** on planes which are <u>not perpendicular</u> to the axis of the bolt or pin.

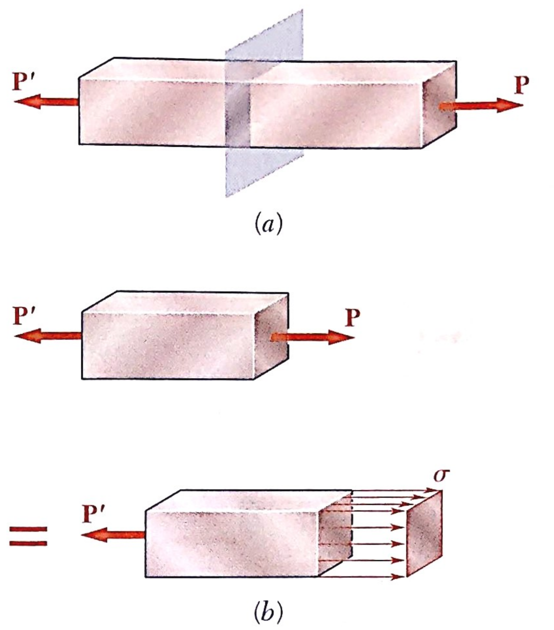


Fig. 1.28

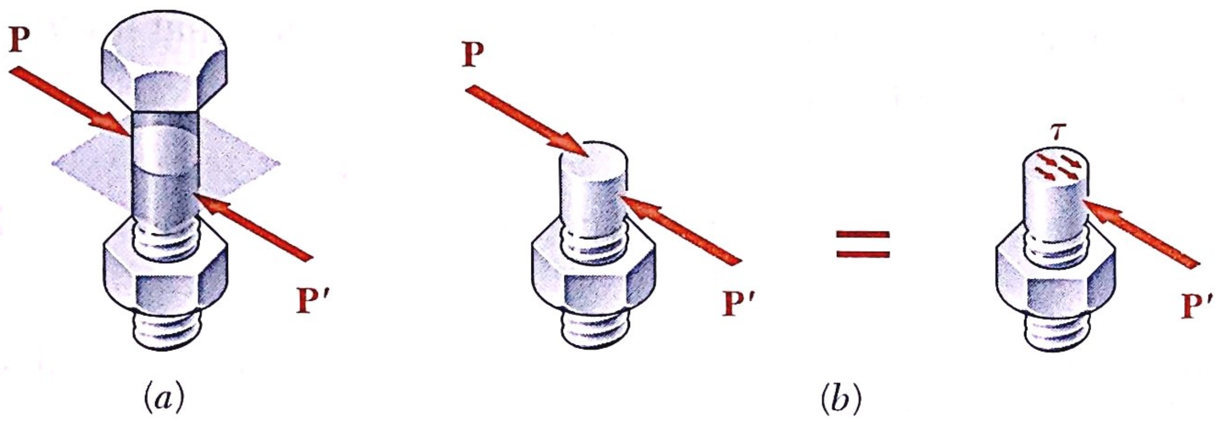


Fig. 1.29

- Consider the two-force member of Fig.28, which is subjected to axial forces **P** and **P’**.
- If we pass a section forming an angle θ with a normal plane (Fig.30a) and draw the free-body diagram of the portion of member located to the left that section (Fig.1.30b),

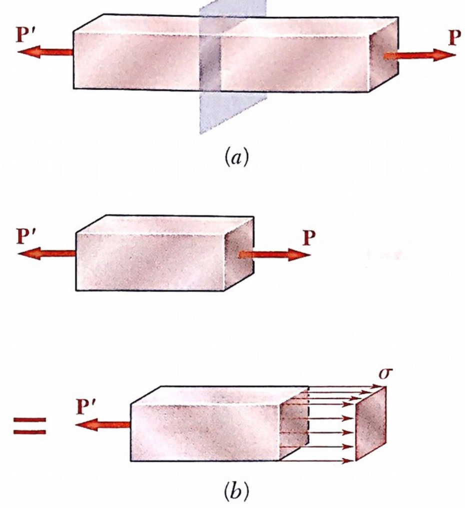



Fig. 1.28

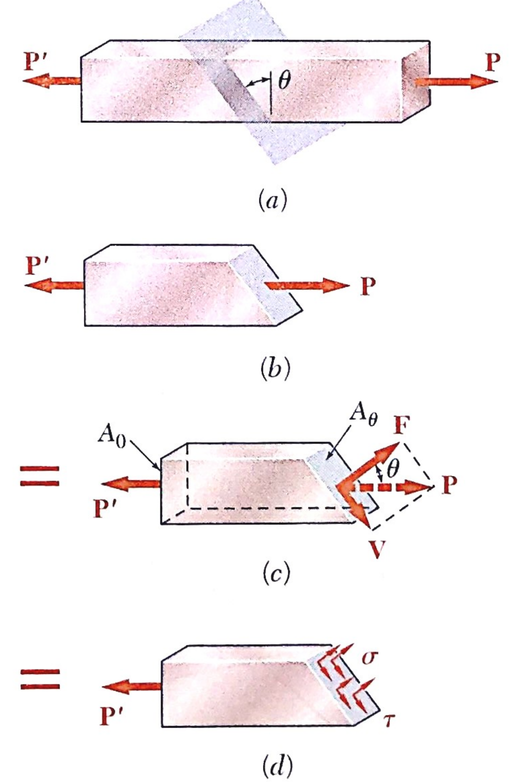



Fig. 1.30

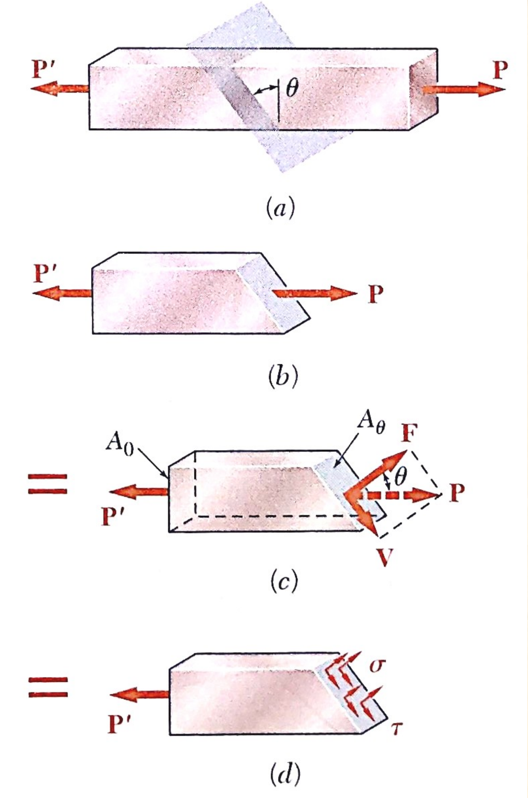


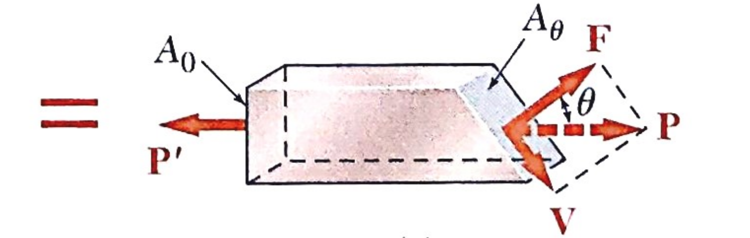
Fig. 1.30

- It would be found from the equilibrium conditions of the free body that the distributed forces acting on the section must be equivalent to the force **P**.

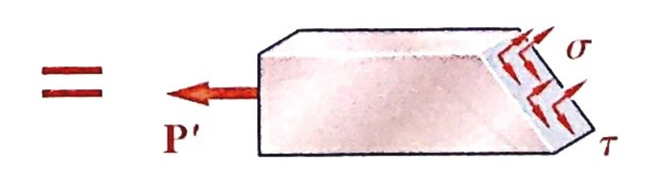
- Resolving **P** into components **F** and **V**, respectively normal and tangential to the section (Fig.1.30c), it should be;

$$F = P\cos\theta \qquad V = P\sin\theta \qquad (1.12)$$

- The force **F** represents the resultant of normal forces distributed over the section, and the force **V** the resultant of shearing forces (Fig.1.30d).



(c)



(d)

Fig. 1.30

- The average values of the corresponding normal and shearing stresses are obtained by dividing, respectively, **F** and **V** by the area $\boldsymbol{A_\theta}$ of the section:

$$\sigma = \frac{F}{A_\theta} \qquad \tau = \frac{V}{A_\theta} \qquad (1.13)$$

- Substituting for F and V from (1.12) into (1.13), and observing from Fig.30c that

- Substituting for **F** and **V** from (1.12) into (1.13), and observing from Fig.30c that

$$A_0 = A_\theta \cos\theta$$

or

$$A_\theta = A_\theta / \cos\theta,$$

where $A_0$ denoted the area of a section perpendicular to the axis of the member.

- These equations will be obtained

$$\sigma = \frac{P\cos\theta}{A_0/\cos\theta} \qquad \tau = \frac{P\sin\theta}{A_0/\cos\theta}$$

or $$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta \qquad (1.14)$$

It should be noted from the first of Eqs.(1.14) that the normal stress ***σ*** is maximum when ***θ=0***, *i.e.*, when the plane of the section is perpendicular to the axis of the member, and that it approaches ***zero*** as ***σ*** approaches ***90°***.

- The value of σ should be checked when σ=0, it should be

(1.15)

as it is found earlier session.

- The second of Eqs,(1.14) shows that the shearing stress τ is zero for σ=0 and σ=90°, and that for σ=45° it reaches its maximum value

$$\tau_m = \frac{P}{A_0} \sin 45^\circ \cos 45^\circ = \frac{P}{2A_0} \qquad (1.16)$$

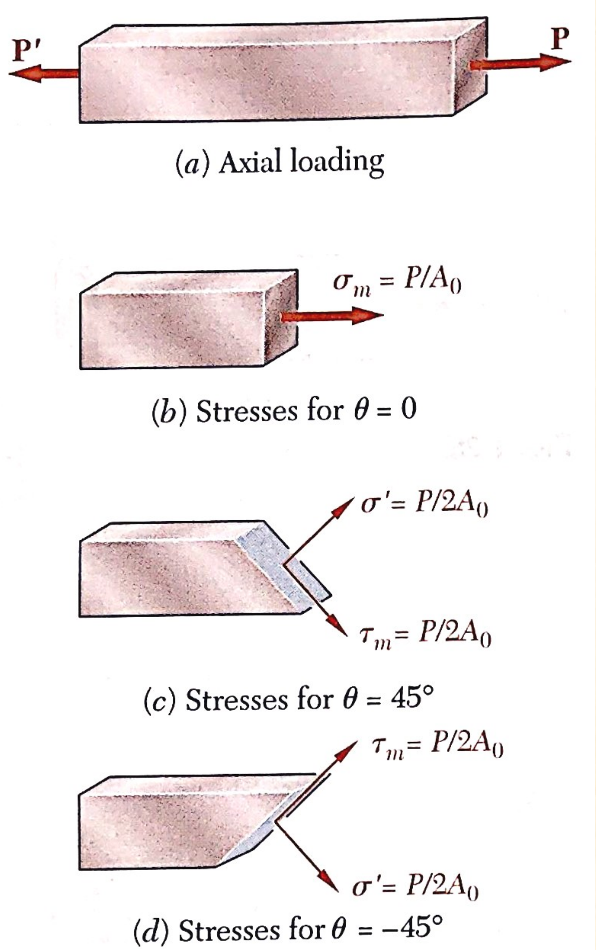



Fig. 1.31

- The first of Eqs.(1.14) indicates that, when σ=45°, the normal stress σ' is also equal to $P/2A_0$:

$$\sigma' = \frac{P}{A_0}\cos^2 45^\circ = \frac{P}{2A_0} \qquad (1.17)$$

- The results obtained in Eqs. (1.15), (1.16), and (1.17) are shown graphically in Fig.1.31.

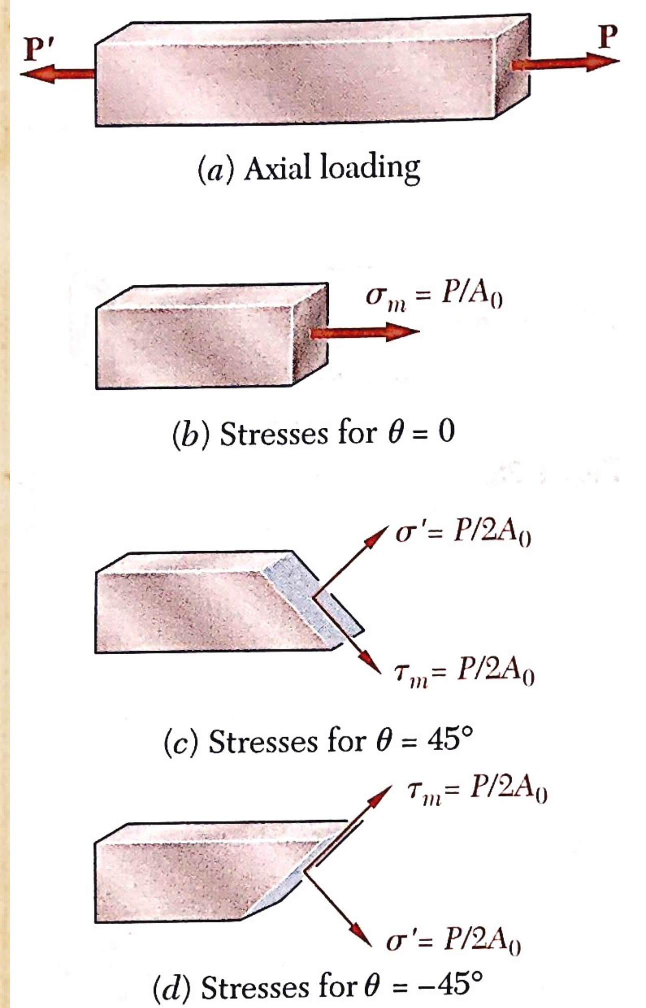


Fig. 1.31

- It is noted that the same loading may produce either a normal stress

$$\sigma_m = P/A_0$$

and no shearing stress (Fig.1.31b), or a normal and shearing stress of the same magnitude

$$\sigma' = \tau_m = P/2A_0$$

(Fig.1.31 *c* and *d*), depending upon the orientation of the section.

# *Question : week9*

- Two wooden members of uniform rectangular cross section are joined by the simple glues scarf splice shown. Knowing that P=11 kN, determine the normal and shearing stresses in the glued splice.