Introduction into design engineering week 6

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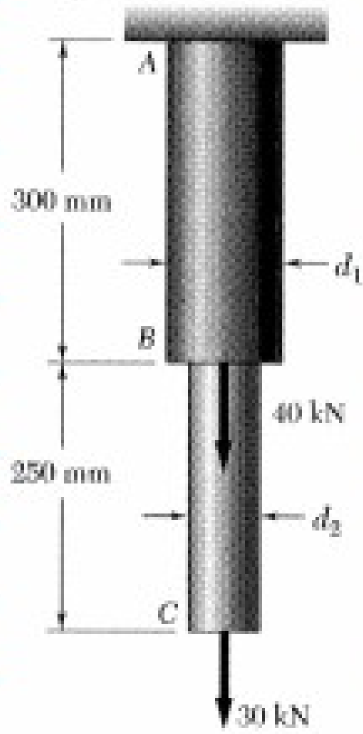
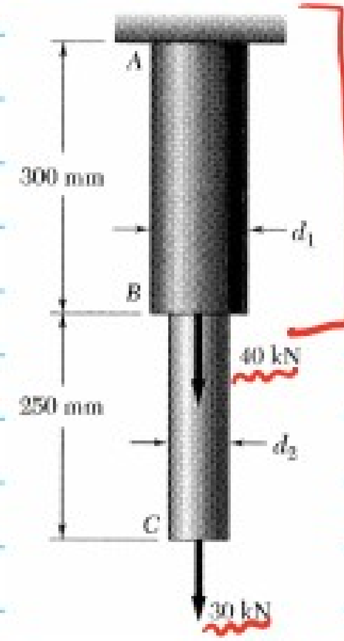
Q3 :



Two solid cylindrical rods AB and BC are welded together at B and loaded as shown.

Knowing that the average normal stress must not exceed 175 MPa in rod AB and 150 MPa in rod BC, determine the smallest allowable values of d_1 and d_2.

[Rod AB]



$$P = 40\,\text{kN} + 30\,\text{kN} = 70\,\text{kN} = 70000\,\text{N}$$

$$\text{Area } AB = \left(\frac{d_1}{2}\right)^2 \pi = \frac{d_1^2}{4}\pi$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{70000}{\frac{d_1^2 \pi}{4}} = \frac{280000}{d_1^2 \pi}$$

Now σ_{AB} is assumed as 175 MPa.

$$175\,\text{MPa} = \frac{280000}{d_1^2 \pi}$$

$$175 \cdot d_1^2 \pi = 280000$$

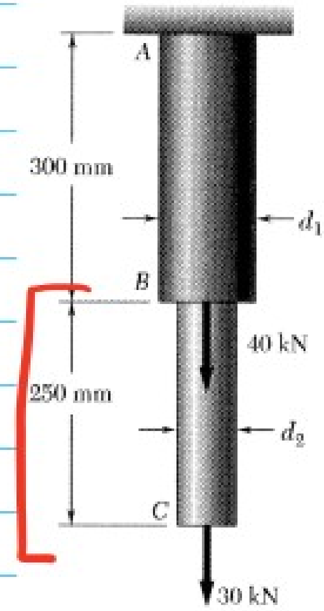
$$d_1 = \sqrt{\frac{280000}{175 \cdot \pi}} = 22.56 \cong \underline{22.6\,\text{mm}}$$

Thus the smallest diameter is $\underline{22.6\,\text{mm}}$.

[Rod BC]

$P = 30\,kN$

$$\text{Area } BC = \left(\frac{d_2}{2}\right)^2 \pi = \frac{d_2^2}{4}\pi$$



$$\sigma_{BC} = \frac{P}{\text{Area}BC} = \frac{30000}{\frac{d_2^2\pi}{4}} = \frac{120000}{d_2^2\pi}$$

Here σ_{BC} is assumed as 150MPa.

$$150\,MPa = \frac{120000}{d_2^2\pi}$$

$$d_2^2\pi \cdot 150 = 120000$$

$$\therefore \; d_2 = \sqrt{\frac{120000}{150\pi}} = 15.96 \approx 16.0\,mm$$

Thus the minimum diameter of Rod BC is 16.0 mm

Application to the analysis and design of simple structure

- It is now in a position to determine the stresses in the members and connections of various simple two-dimensional structures and , thus, to design such structures.

- As an example, let us return to the structure of Fig.1.1 that is had been already considered in previous classes.
- Now, let us specify the supports and connections at *A*, *B*, and *C*.

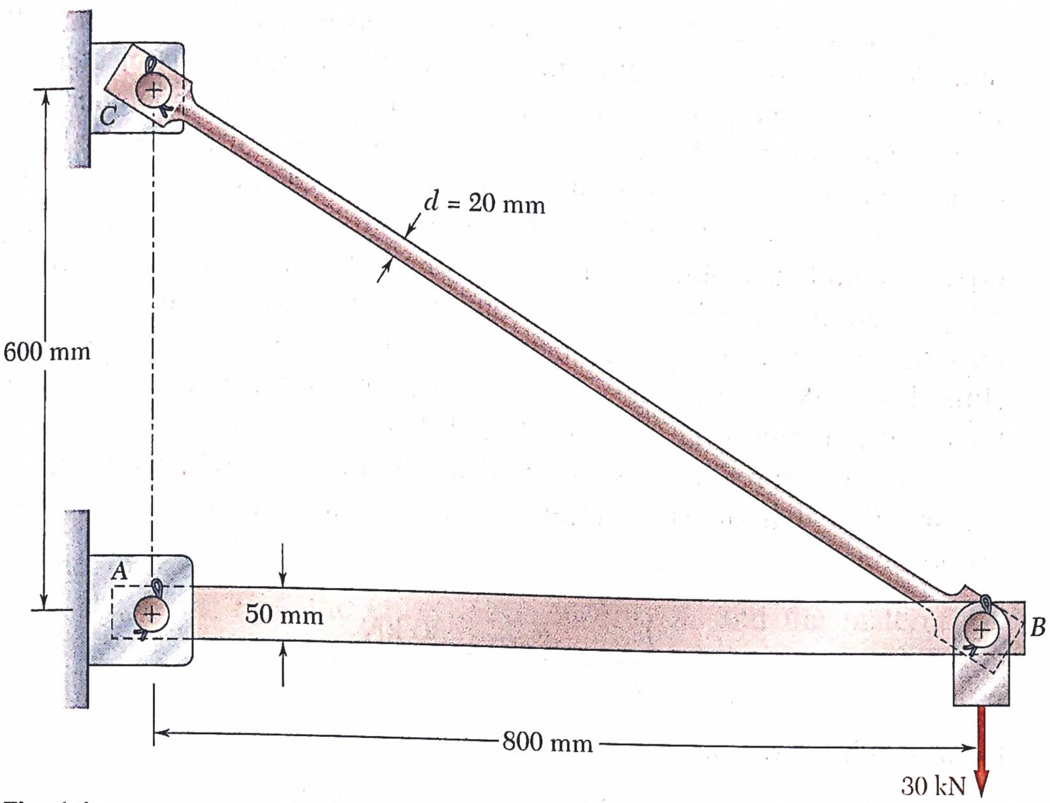


Fig. 1.1

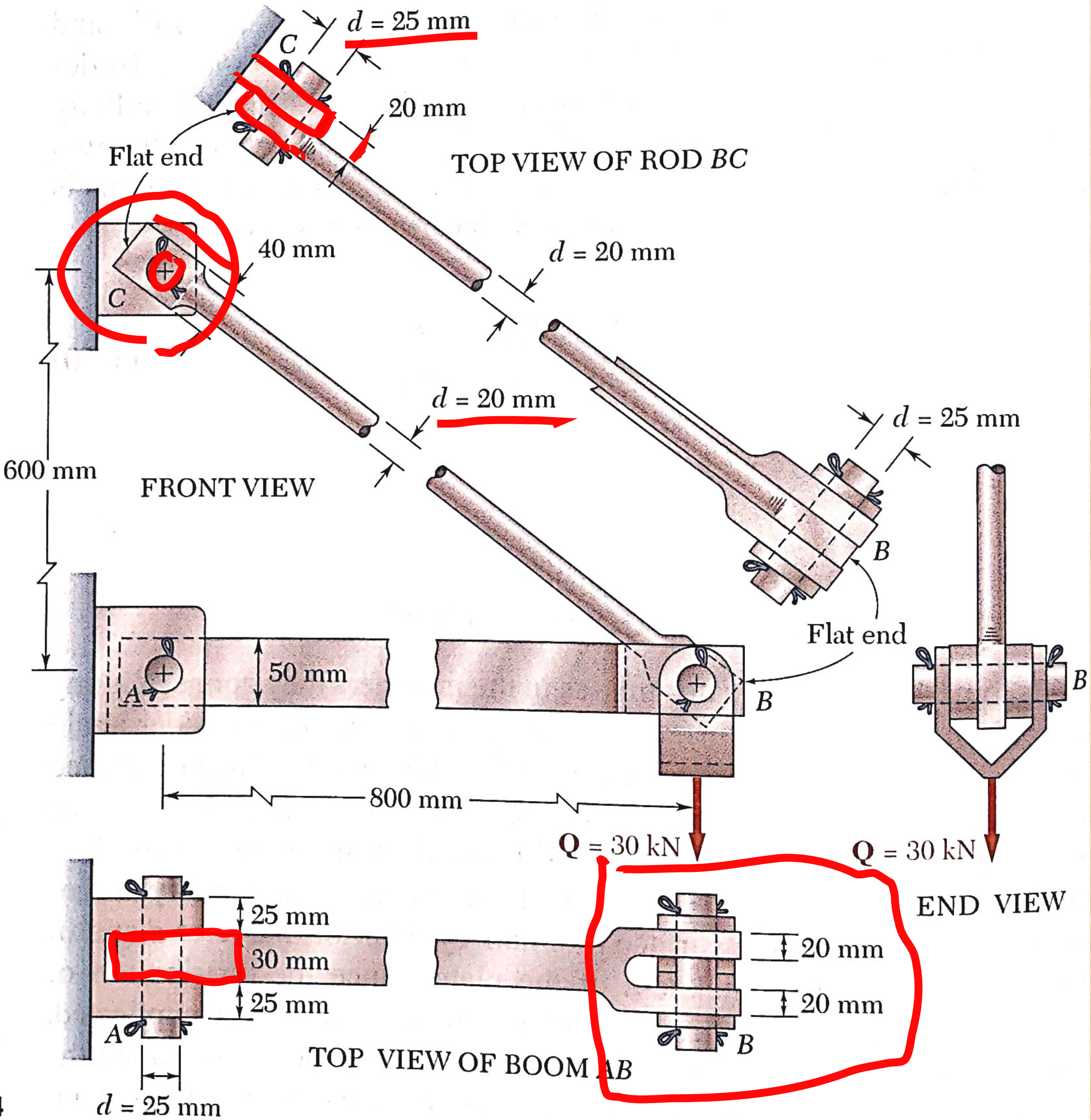


Fig. 1.24

- As shows in Fig.1.24, the 20-mm-diameter rod *BC* has flat ends of 20 × 40 -mm rectangular cross section, while boom *AB* has a 30 × 50-mm rectangular cross section and is fitted with a clevis at end *B*.

- Both members are connected at *B* by pin from which the 30-kN load is suspended by means of a U-shaped bracket.
- Boom *AB* is supported at *A* by a pin fitted into a double bracket, while rod *BC* is connected at *C* to a single bracket.
- All pins are 25 mm in diameter.

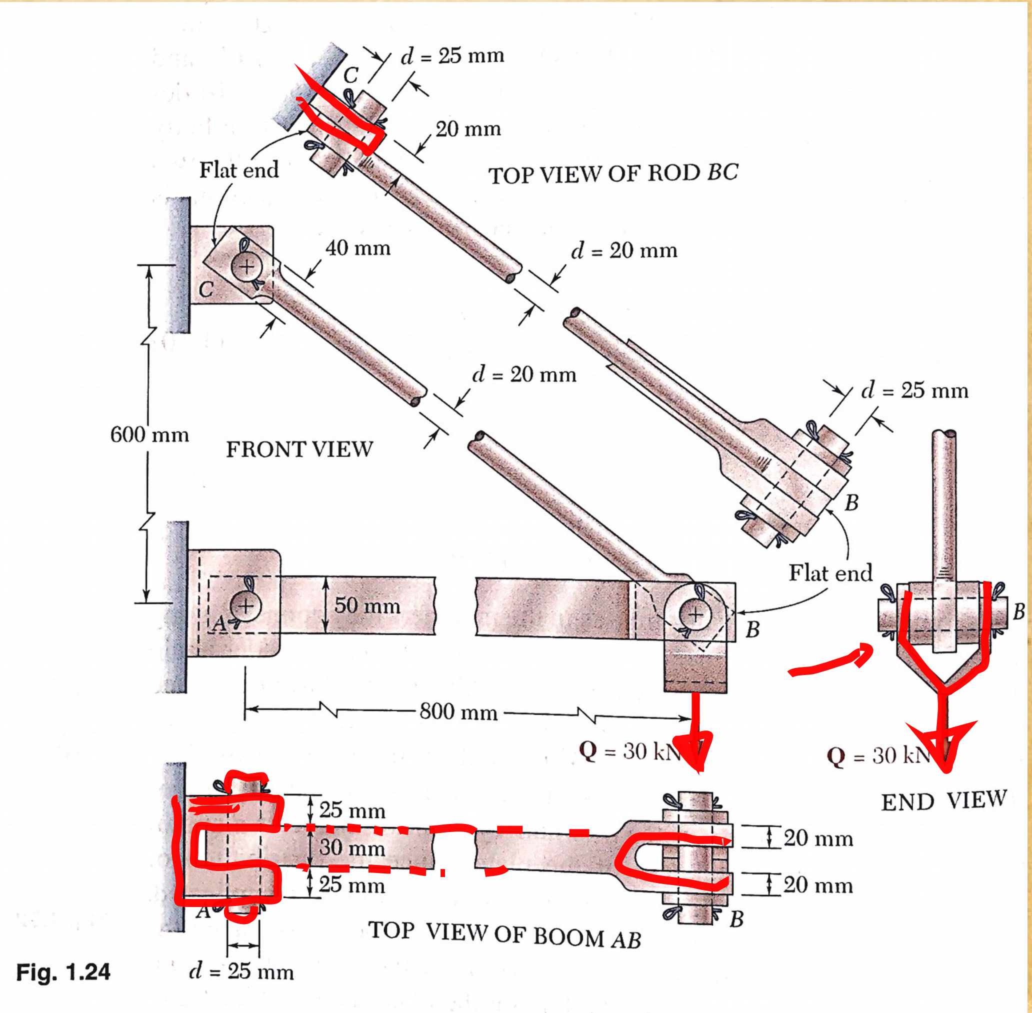


Fig. 1.24

Question A:

Please determine the normal stress in Boom AB and rod BC.

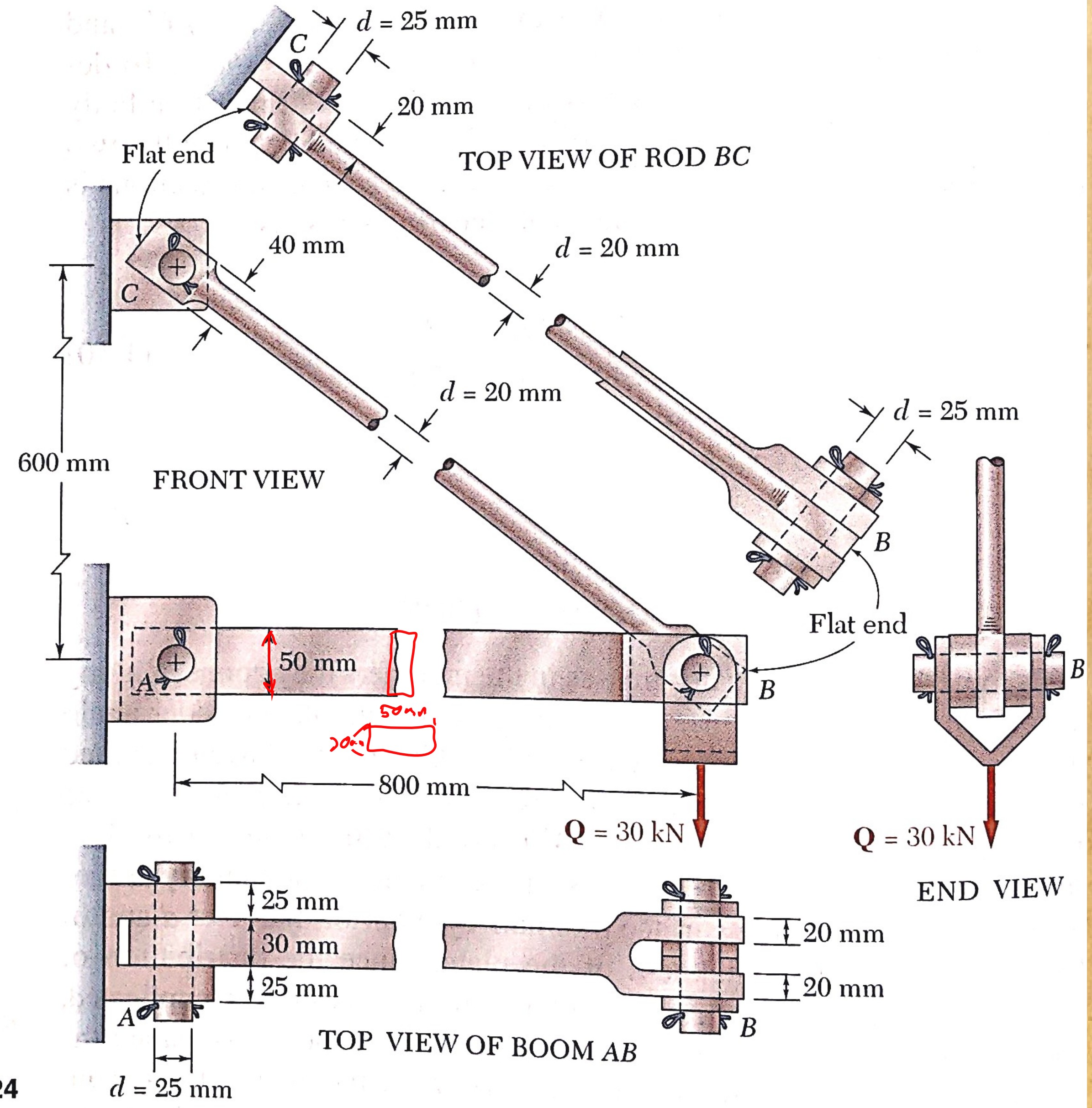
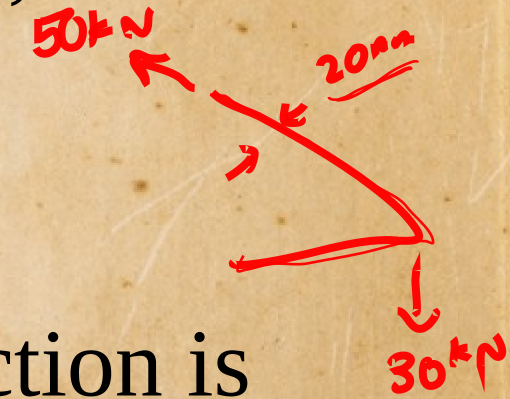


Fig. 1.24

Step 1: forces and stresses

- As it was found in previous lessons, the force in rod *BC* is

$$\underline{\boldsymbol{F_{BC}=50\ kN}}\ \text{(tension)}$$



and the area of its circular cross section is

$$\underline{\boldsymbol{A = 314 \times 10^{-6}\ m^2}}$$

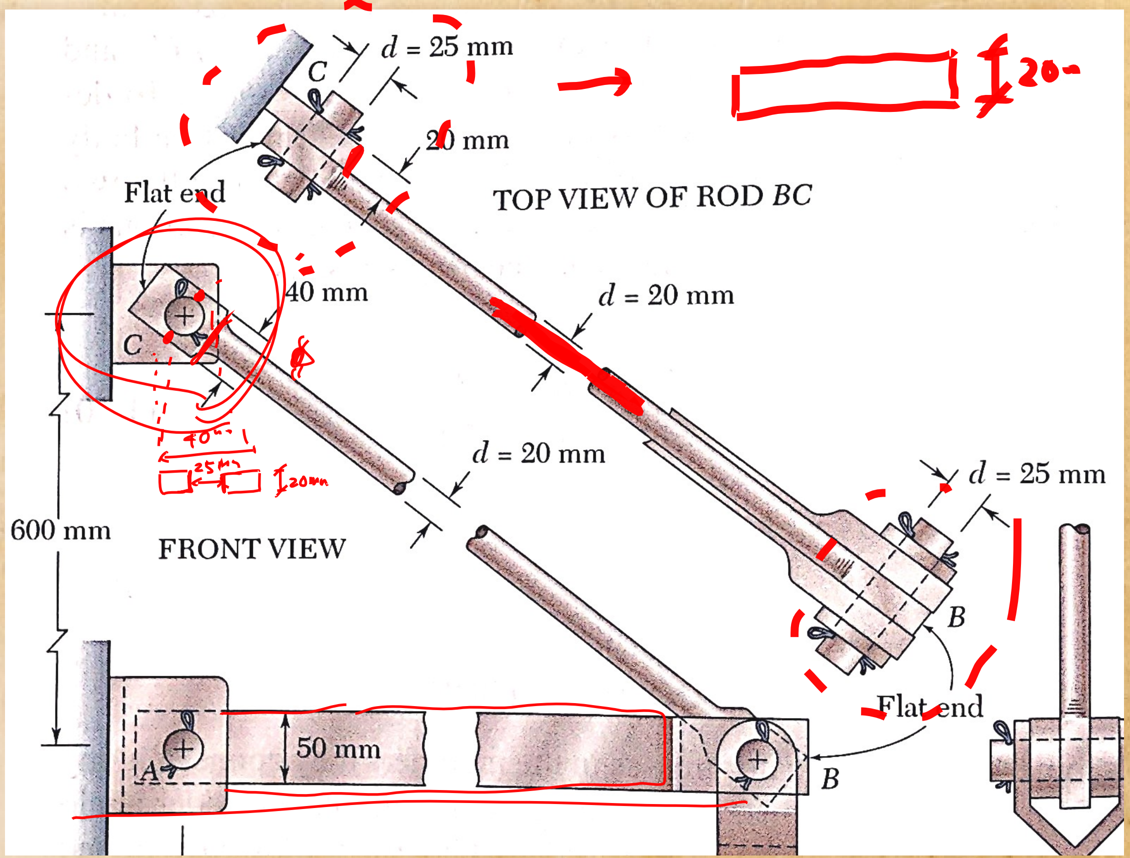
$\sigma = \frac{P}{A}$

; the corresponding average normal stress is

$$\underline{\boldsymbol{\sigma_{BC} = +159\ \text{MPa}.}}$$

Step 2: calculate separate parts

- However, the flat parts of the rod are also under tension and at the narrowest section, where a hole is located, the area is;



$$A = (20\ \text{mm})(40\ \text{mm} - 25\ \text{mm}) = 300 \times 10^{-6}\ \text{m}^2$$

Step 3: find average stress

160 MPa

- The corresponding ***average value of the stress,*** therefore, is

$$(\sigma_{BC})_{\text{end}} = \frac{P}{A} = \frac{\boxed{50 \times 10^3 \text{ N}}}{300 \times 10^{-6} \text{ m}^2} = \boxed{167 \text{ MPa}}$$

167 MPa

159 MPa

- Note that this is ***an average value***; close to the hole, the stress will actually reach a much larger value.

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- It is clear that, under an increasing load, the rod will fail near one of the holes rather than in its cylindrical portion; its design, therefore, could be improved by *increasing the width or the thickness of the flat ends* of the rod.

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Step 4: Find AB

- Turning now our attention to boom ***AB***, it is recalled from the previous lesson, that the force in the boom is $\boldsymbol{F_{AB}=40\ kN}$ (compression).

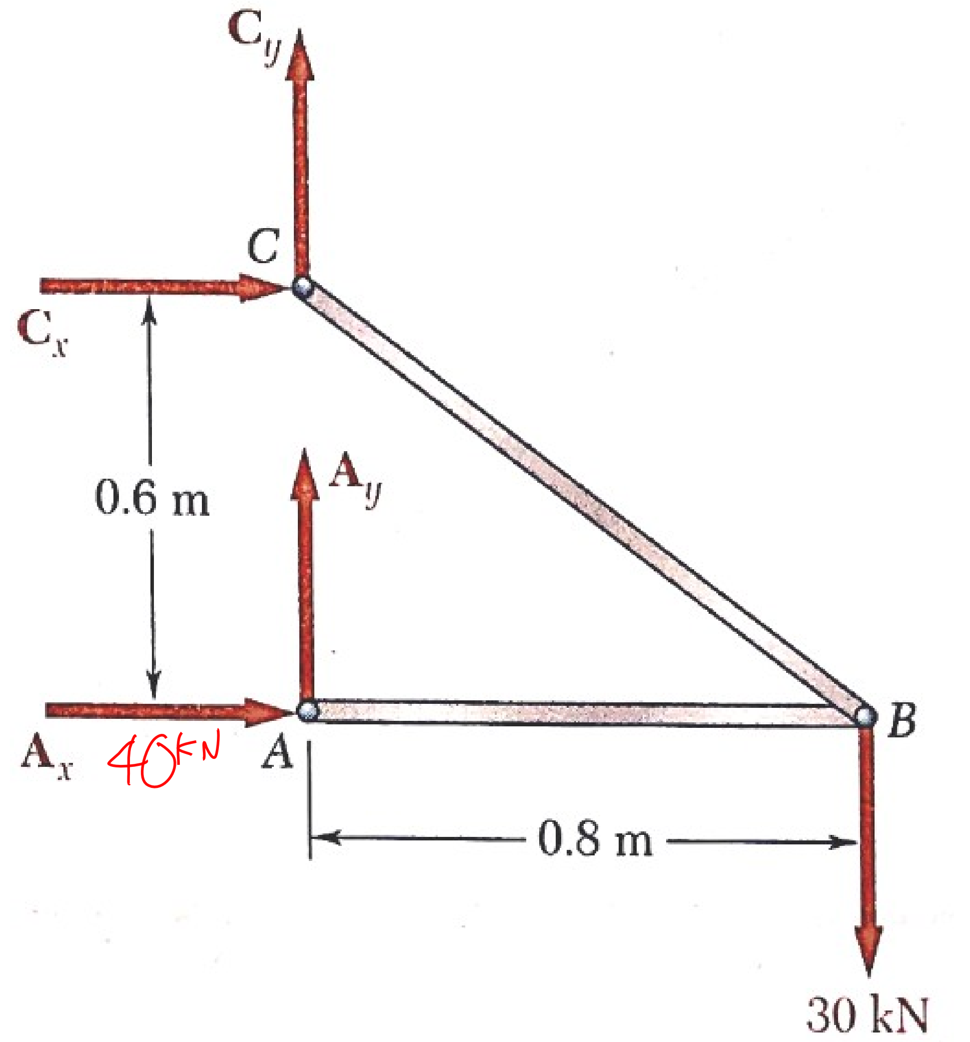


Fig. 1.2

Step 5: Find area and stress

- Since the area of the boom's rectangular cross section is

50mm

30mm

A = 30 mm × 50 mm = 1.5 x 10 $^{-3}$ ***m*** ^{2}***,***

the *average value of the normal stress* in the main part of the rod, between pins ***A*** and ***B***, is

$$\sigma_{AB} = -\frac{40 \times 10^3 \text{ N}}{1.5 \times 10^{-3} \text{ m}^2} = -26.7 \times 10^6 \text{ Pa} \boxed{= -26.7 \text{ MPa}}$$

- Note that the sections of minimum area at ***A*** and ***B*** are not under stress, since the boom is in compression, and, therefore, *pushes* on the pins (instead of pulling on the pins as rod ***BC*** does).

Application to the analysis and design of simple structure

- It is now in a position to determine the stresses in the members and connections of various simple two-dimensional structures and , thus, to design such structures.

Question B:

Please determine of the shearing stress in various connections.

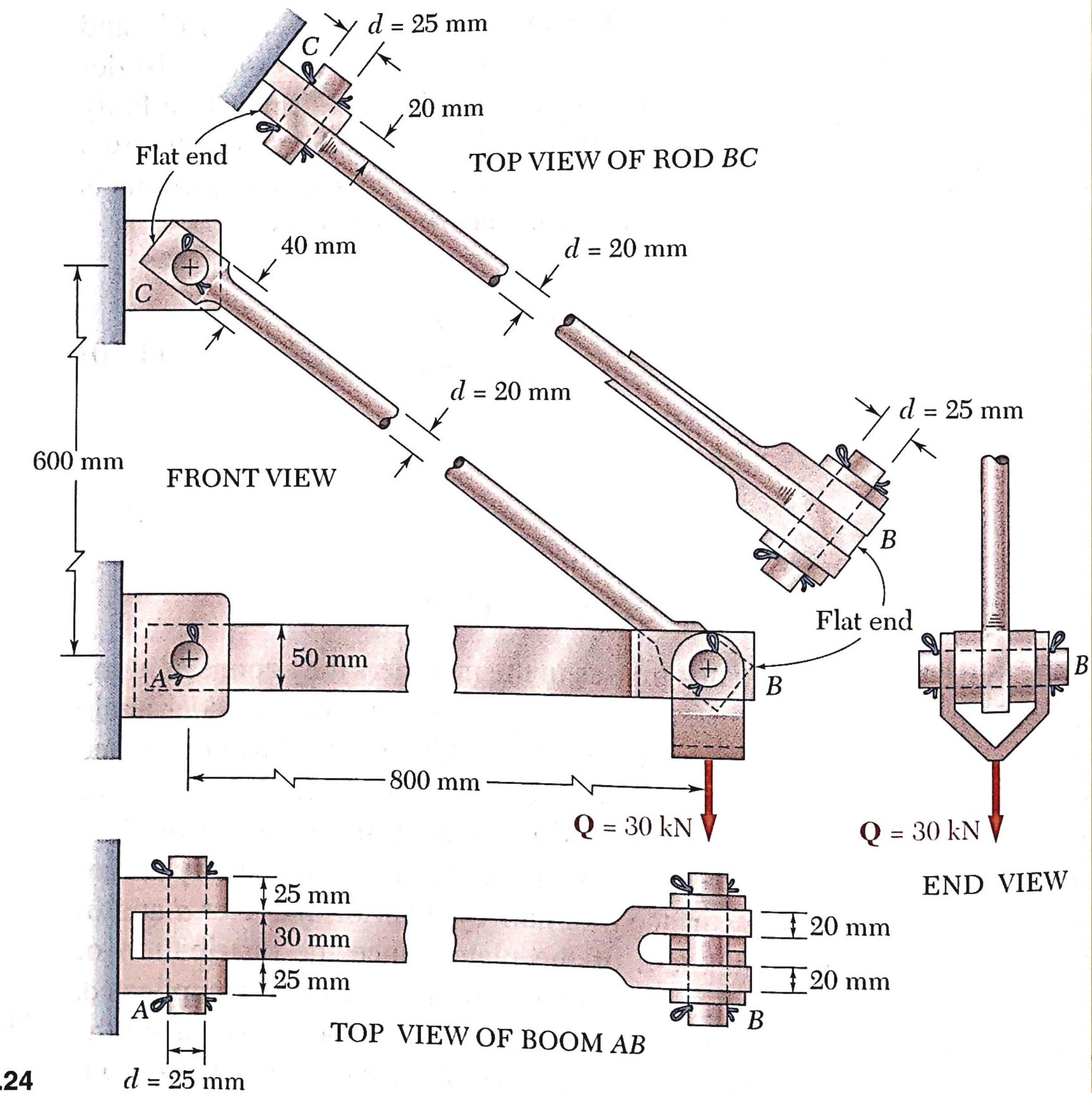


Fig. 1.24

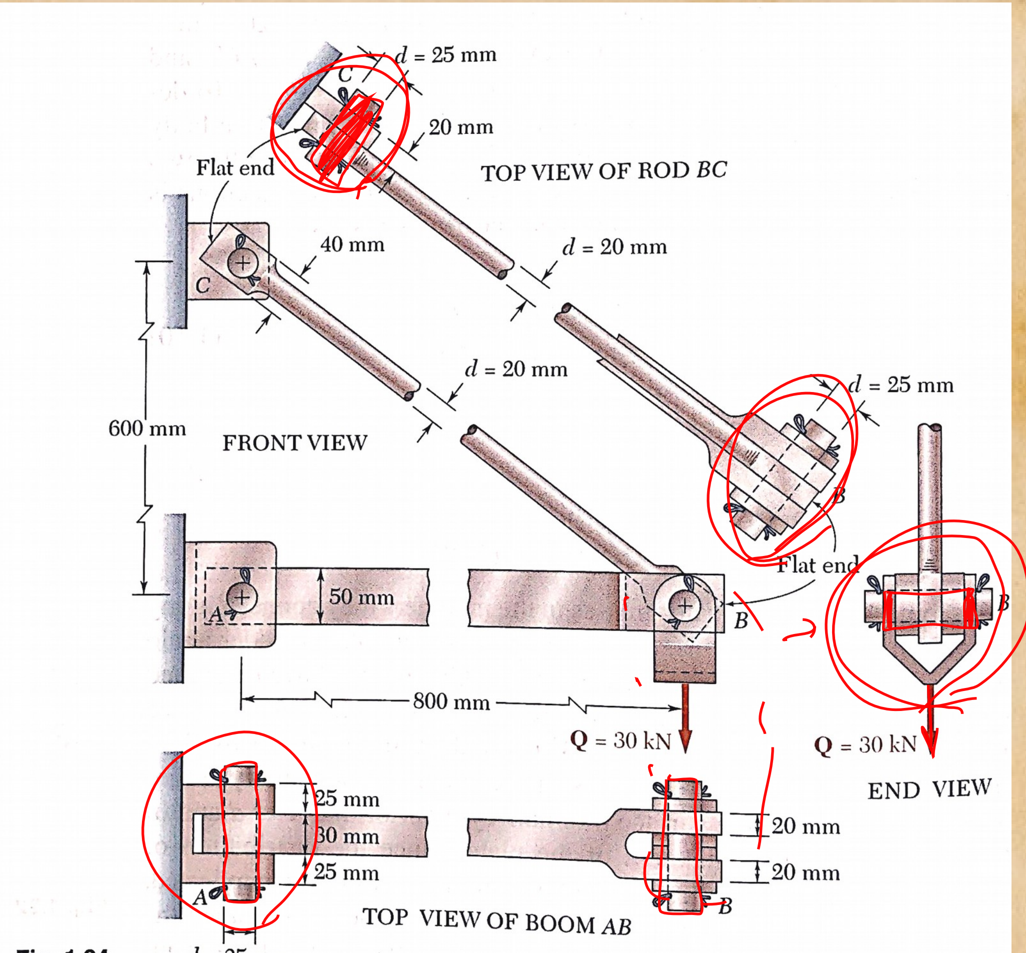


Fig. 1.24

- To determine the shearing stress in a connection such as a bolt, pin, or rivet, it is clearly shown the forces exerted by the various members it connects.

Step 1: draw around pin C

- Thus, in the case of pin ***C*** of this example (Fig.*1.25a*), it can be drawn Fig.*1.25b*, showing the ***50-kN*** force exerted by member ***BC*** on the pin, and the equal and opposite force exerted by the bracket.

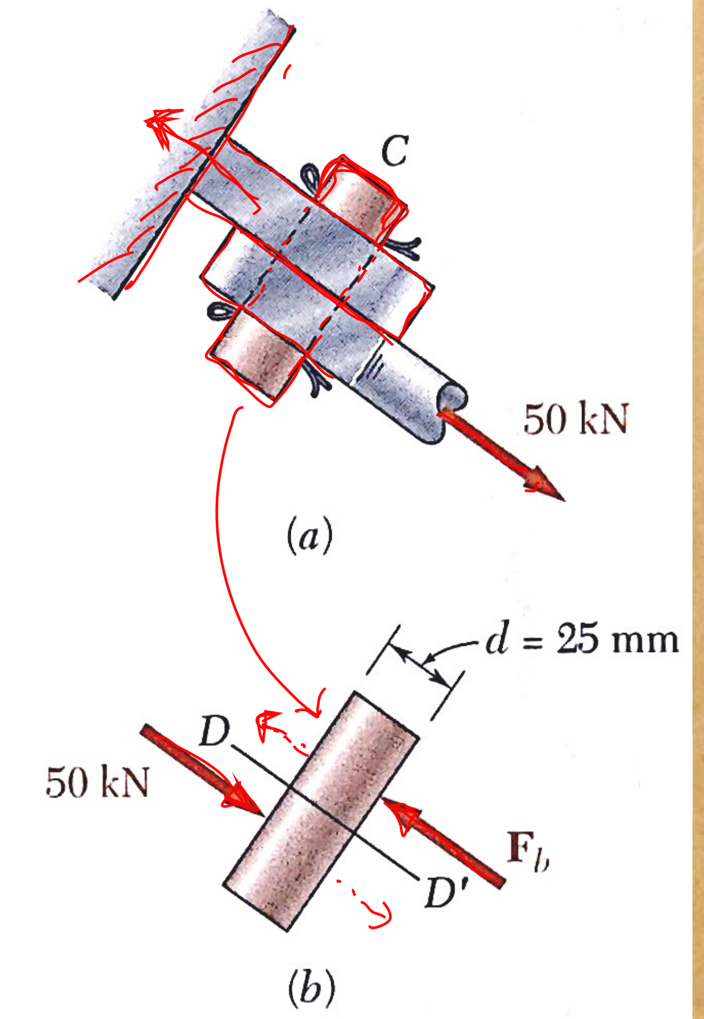


Fig. 1.25

Step 2: draw diagram around C

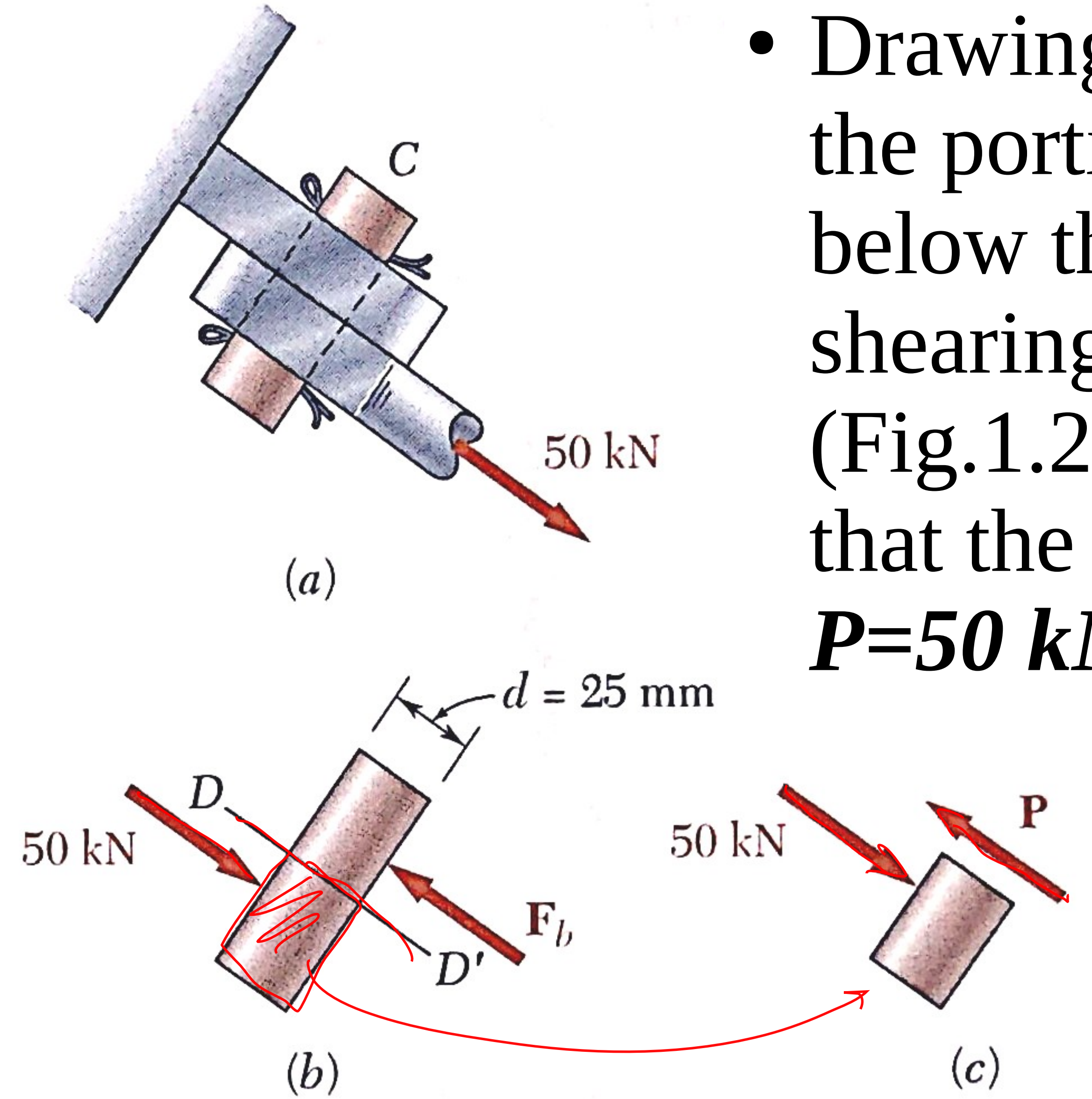


Fig. 1.25

- Drawing now the diagram of the portion of the pin located below the plane DD' where shearing stresses occur (Fig.1.25c), it is concluded that the shear in that plane is ***P=50 kN*.**

Step 3: Calculate Area and stress

- Since the cross sectional area of the pin is

$$A = \pi r^2 = \pi\left(\frac{25\text{ mm}}{2}\right)^2 = \pi(12.5 \times 10^{-3}\text{ m})^2 = 491 \times 10^{-6}\text{ m}^2$$

- It is now found that *<u>the average value of the shearing stress</u>* in the pin at **C** is

$$\tau_{\text{ave}} = \boxed{\frac{P}{A}} = \frac{50 \times 10^3\text{ N}}{491 \times 10^{-6}\text{ m}^2} = 102\text{ MPa}$$

Step 4: Consider around pin A

- Considering now the pin at A (Fig.1.26), it is noted that it is in double shear.
- Let’s draw the free-body diagrams of the pin and of the portion of pin located between the planes ***DD’*** and ***EE’*** where shearing stress occur,

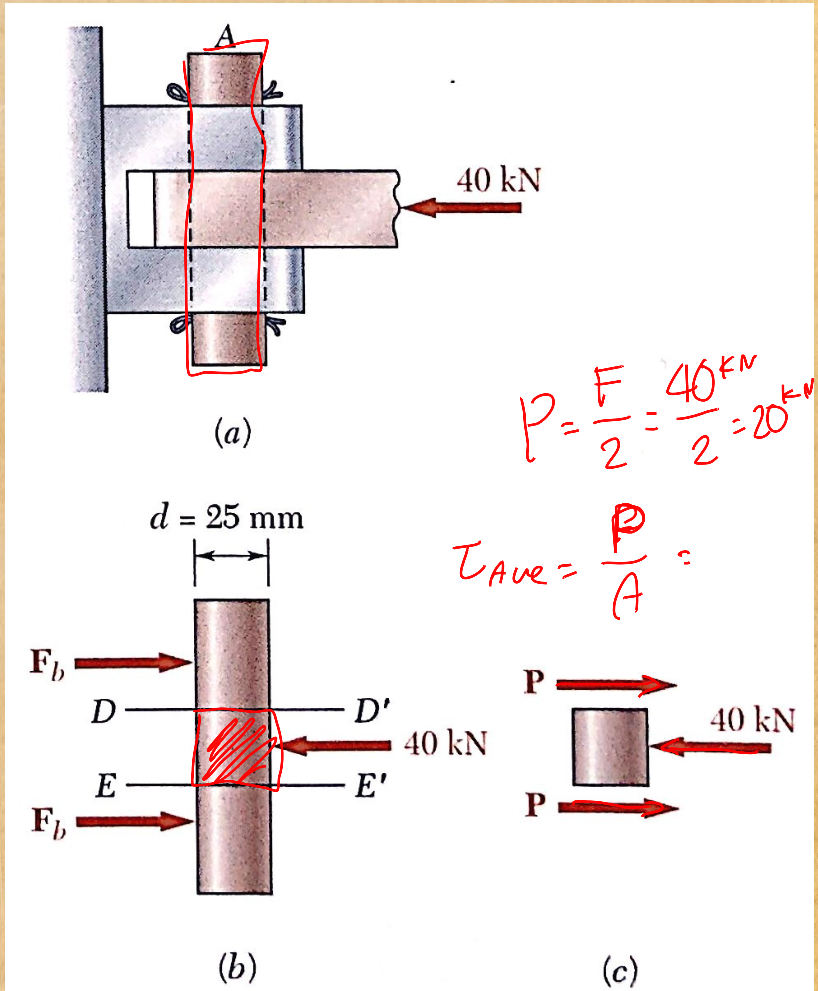


Fig. 1.26

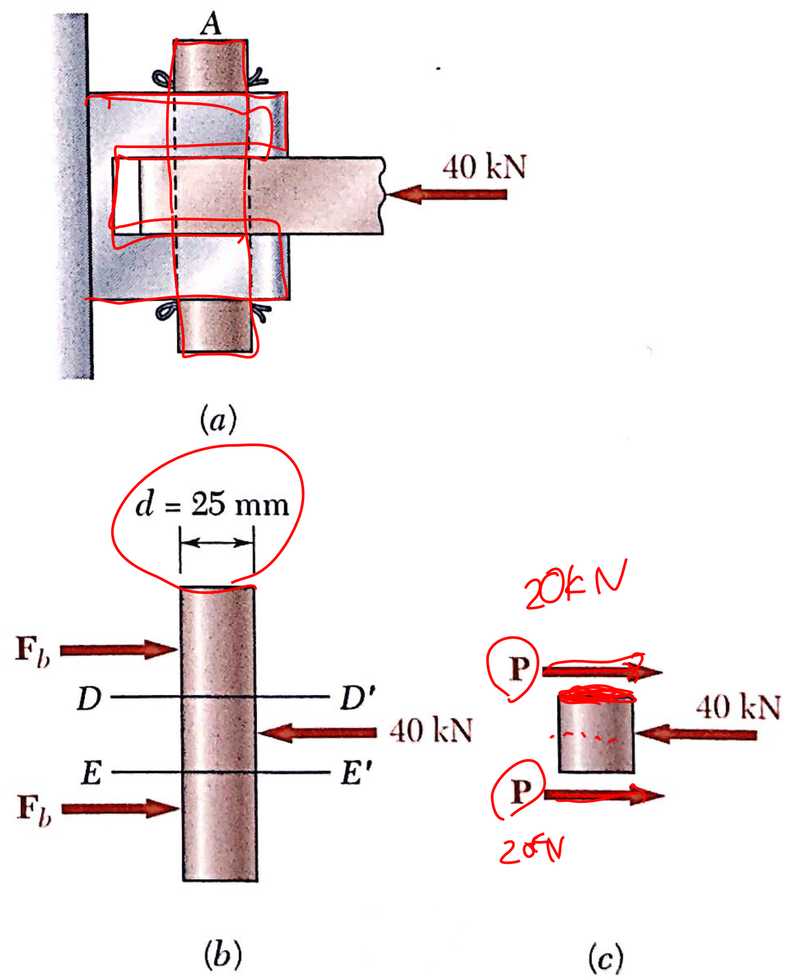


Fig. 1.26

- It is concluded that

$$P=20\ kN$$

P=40kN

P= 20kN

and that

$$\tau_{ave} = \frac{P}{A} = \frac{20\ \text{kN}}{491 \times 10^{-6}\ \text{m}^2} = \boxed{40.7\ \text{MPa}}$$

$$A = \left(\frac{25}{2}\right)^2 \pi$$

Step 5: Consider around pin B

- Considering the pin at B (Fig.1.27a), it is noted that the pin may be divided into five portions which are acted upon by forces exerted by the boom, rod, and bracket.

A B

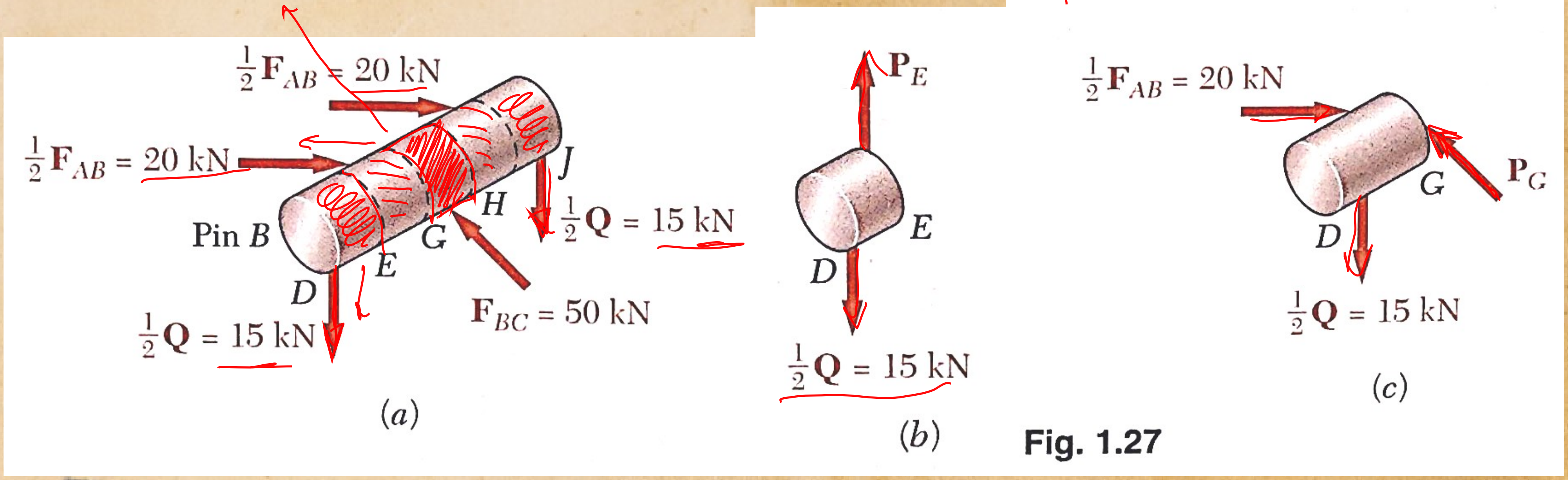


Fig. 1.27

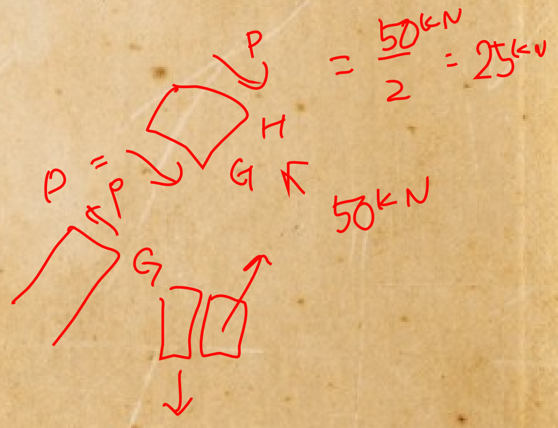
Step 6: Find forces at each parts

- Considering successively the portions ***DE*** (*Fig.1.27b*) and ***DG*** (*Fig.1.27c*), it is concluded that the shear in section ***E*** is

$$\underline{\boldsymbol{P_E = 15\ kN}},$$

while the shear in section ***G*** is

$$\underline{\boldsymbol{P_G = 25\ kN.}}$$



d = 25 mm

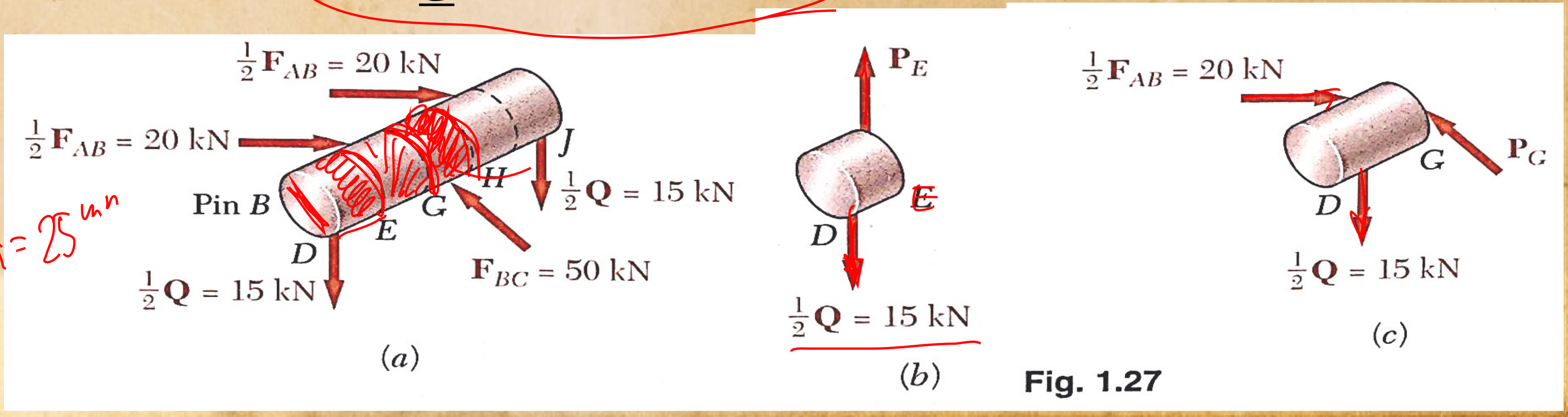


Fig. 1.27

Step 7: Find the stress

- Since the loading of the pin is symmetric, it is concluded that the *maximum value of the shear* in pin ***B*** is

$$\underline{P_G = 25\ kN,}$$

and that the *largest shearing stresses* occur in sections ***G*** and ***H***, where

$$\tau_{ave} = \frac{P_G}{A} = \frac{25\text{ kN}}{491 \times 10^{-6}\text{ m}^2} = \boxed{50.9\text{ MPa}}$$

$\left(\frac{25}{2}\right)^2 \pi$

Question C:

Determination of the bearing Stresses.

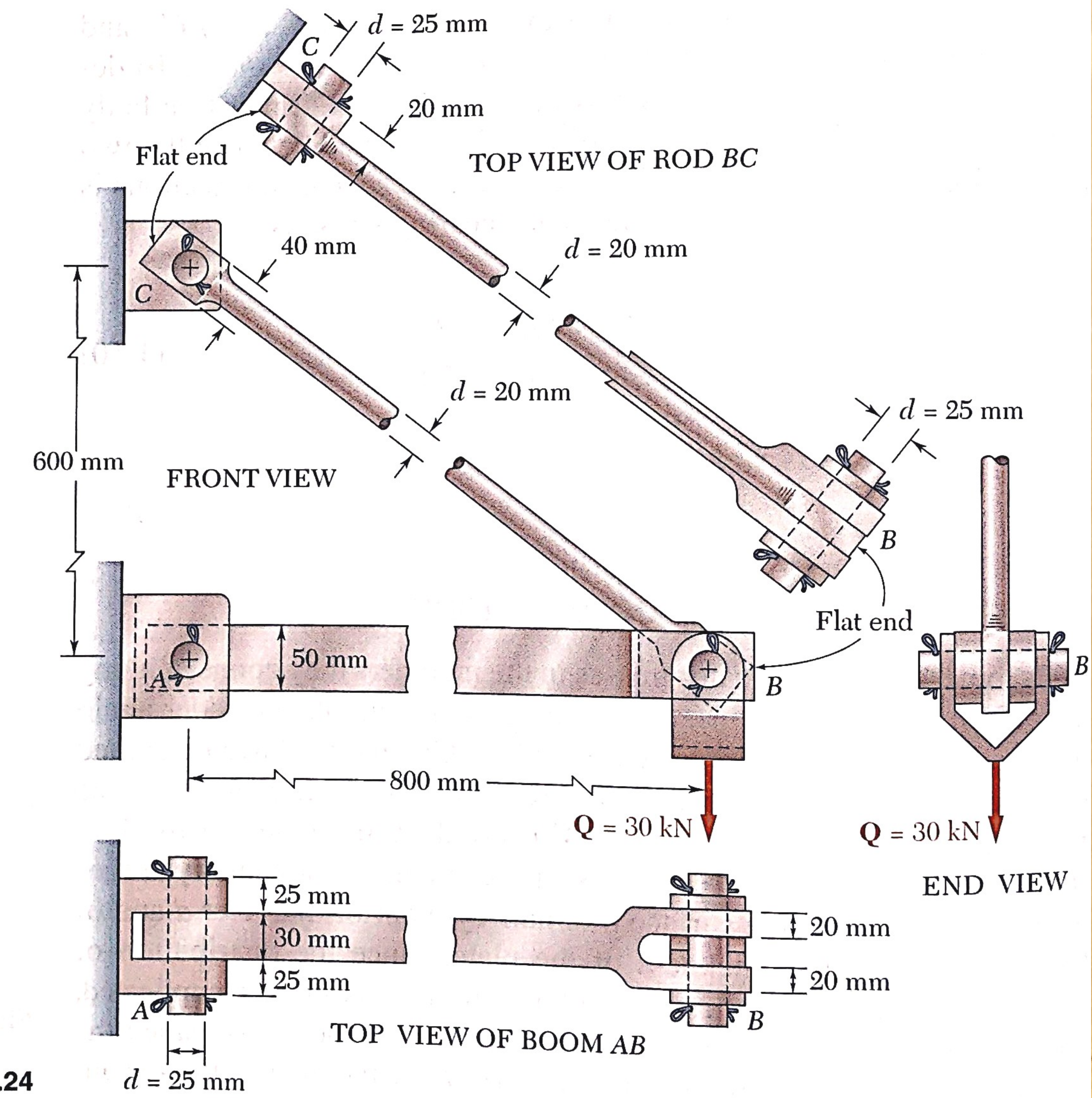


Fig. 1.24

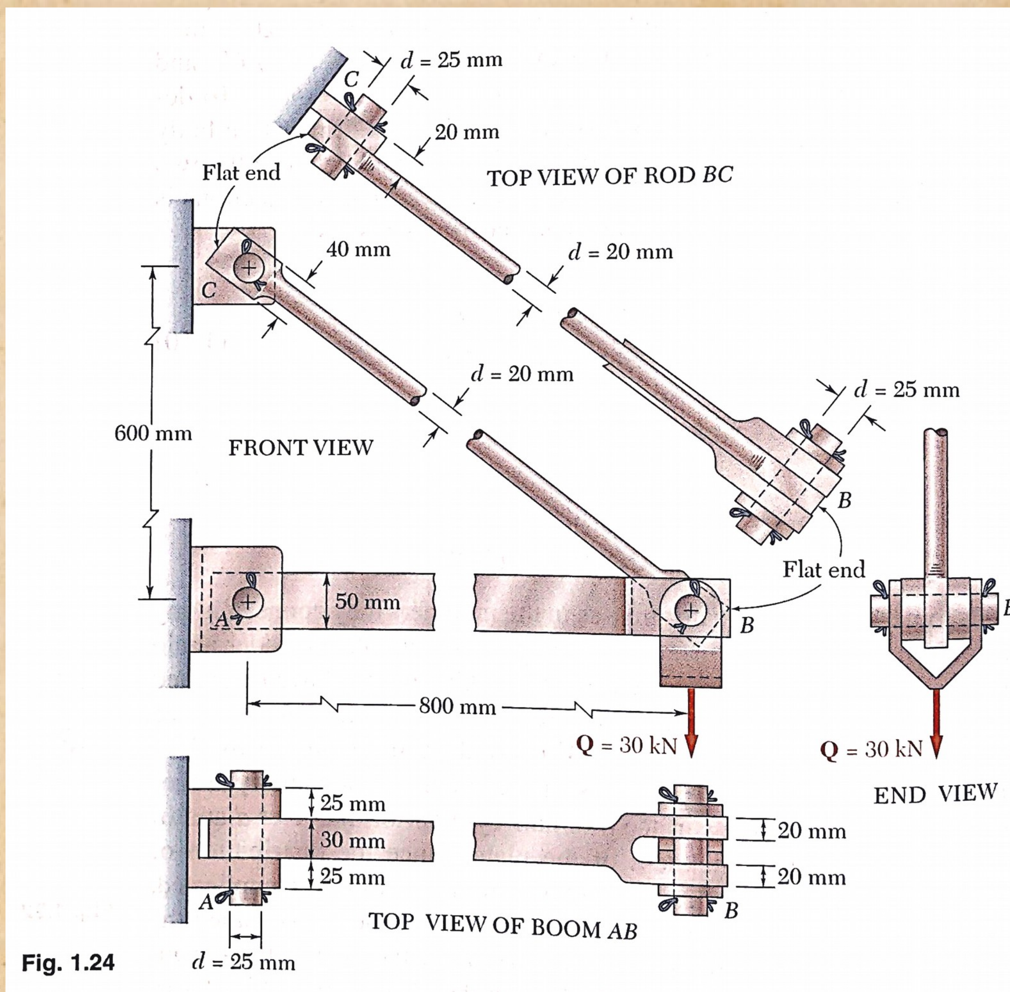


Fig. 1.24

- To determine the norminal bearing stress at a in member AB,

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

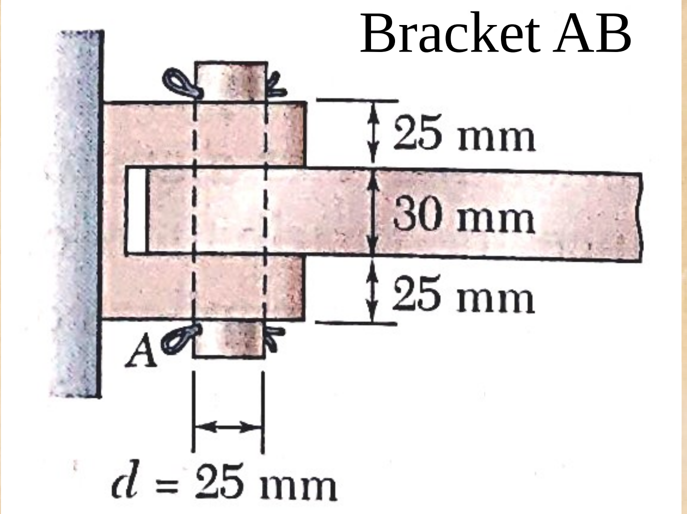
this equation should be used.

- Here $t = 30\ mm$, $d = 25\ mm$, and

$$\underline{\boldsymbol{F = F_{AB} = 40\ kN,}}$$

therefor $\boldsymbol{\sigma_b}$ is;

$$\sigma_b = \frac{P}{td} = \frac{40\text{ kN}}{(30\text{ mm})(25\text{ mm})} = 53.3\text{ MPa}$$



- To obtain the bearing stress in the bracket at A, it should be used that

  ***t* = 2(25mm) = 50 mm**

  ***d* = 25 mm**: thus

$$\sigma_b = \frac{P}{td} = \frac{40\ \text{kN}}{(50\ \text{mm})(25\ \text{mm})} = 32.0\ \text{MPa}$$

- The bearing stress in member *AB*, at *B* and *C* in member *BC*, and in the bracket at *C* are found in a similar way.

Mini quiz 6

When the force P reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line.

Determine the average shearing stress along that surface at the time of failure.