Introduction into design engineering week 5

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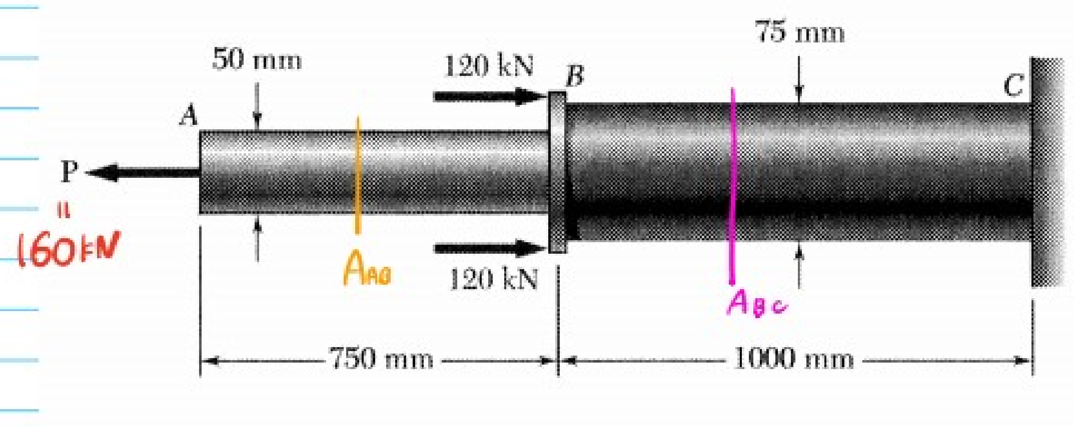
Question 2:

Knowing that $P = 160$ kN, determine the average normal stress at the midsection of (a) rod AB, (b) rod BC.

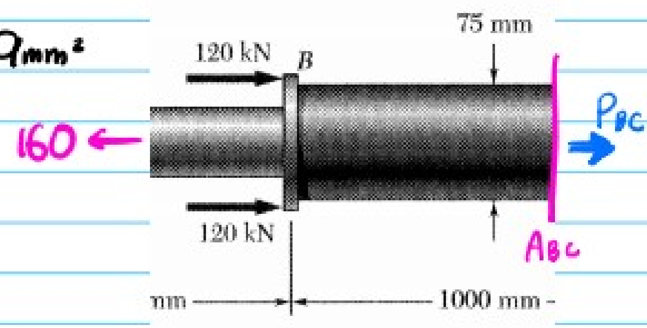


$A_{AB} = 1963.5\ mm^2$

$\sigma_{AB} = \dfrac{P}{A_{AB}} = \dfrac{160kN}{1963.5\,mm^2} = \underline{81.5\ MPa}$ // ← average normal stress at rod AB

(tension)

$A_{BC} = 4417.9\,mm^2$



$P_{BC} + 120 + 120 = 160$

$P_{BC} = 160 - 240$

$= \underline{-80kN}$

$\sigma_{BC} = \dfrac{P_{BC}}{A_{BC}}$

$= \dfrac{-80kN}{4417.9\,mm^2}$

$= -18.1\ MPa$ ← the average normal stress at rod BC.

(compression

Shearing stress

- The internal forces and the corresponding stresses discussed in the previous sessions were normal to the section considered.
- A very different type of stress is obtained when transverse forces P and P’ are applied to a member AB (Fig.1.15)

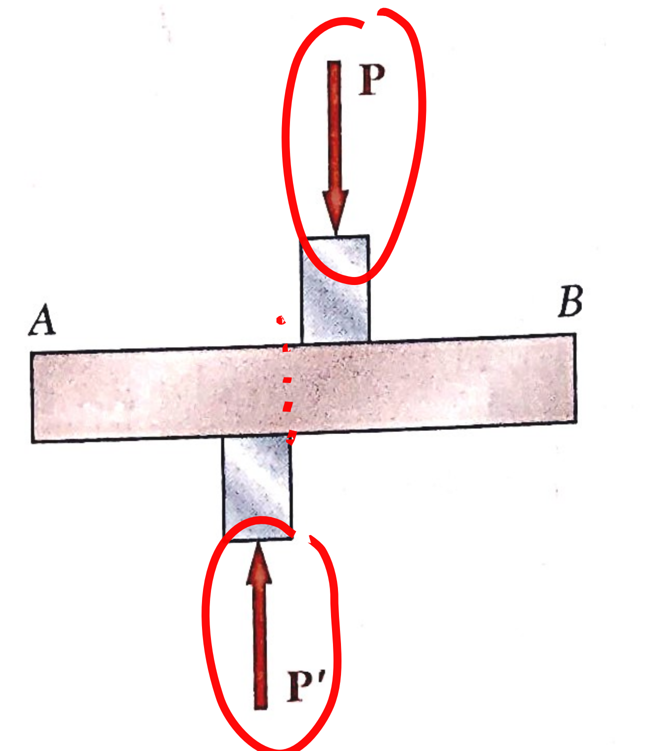


Fig. 1.15

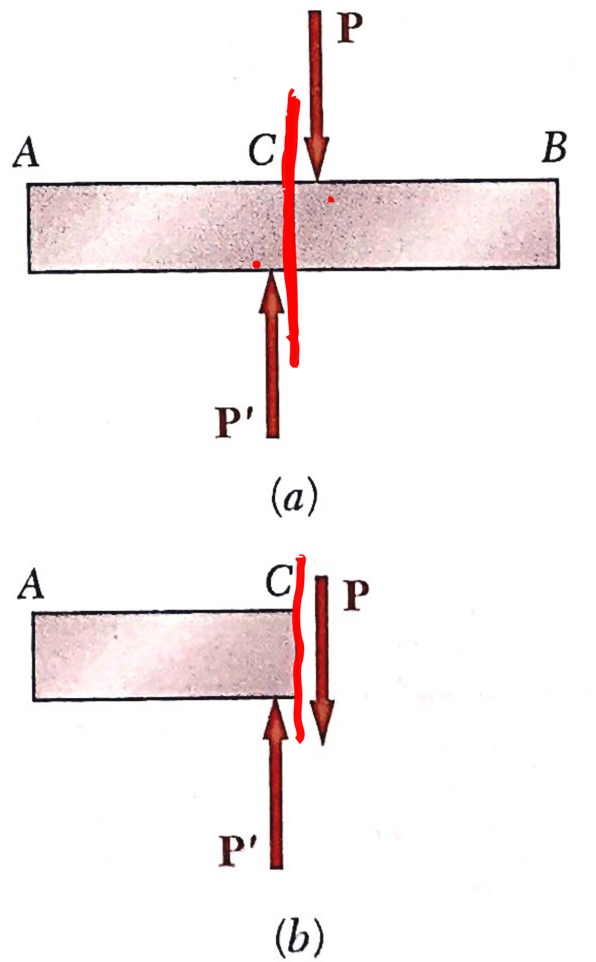
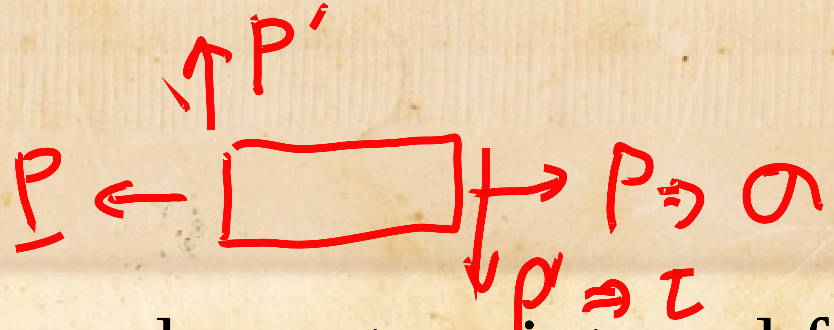


Fig. 1.16

- Passing a section at C between the points of application of the two forces (Fig.1.16*a*), it is obtained that the diagram of portion *AC* shown in Fig.1.16*b*.
- It is concluded that the internal forces must exist in the plane of the section, and that their resultant is equal to ***P***.



$\sigma = \frac{P}{A}$

- These elementary internal forces are called ***shearing forces***, and the magnitude ***P*** of their resultant is the *shear* in the section.
- Dividing the shear ***P*** by the area ***A*** of the cross section, it is obtained the ***average shearing stress*** in the section.
- Denoting the shearing stress by the Greek letter τ (tau), it is written as;

$$\tau_{ave} = \boxed{\frac{P}{A}} \qquad (1.8)$$

$$\boxed{\tau_{ave} = \frac{P}{A}}$$

- It should be emphasized that the value obtained is an ***average value of the shearing stress*** over the entire section.
- Contrary to what we said earlier for normal stresses, the distribution of shearing stresses across the section cannot be assumed uniform.
- The actual value **τ** of the shearing stress varies from zero at the surface of the member to a maximum value τ_{max} that may be much larger than the average value τ_{ave}.

- Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components (Fig 1.17).

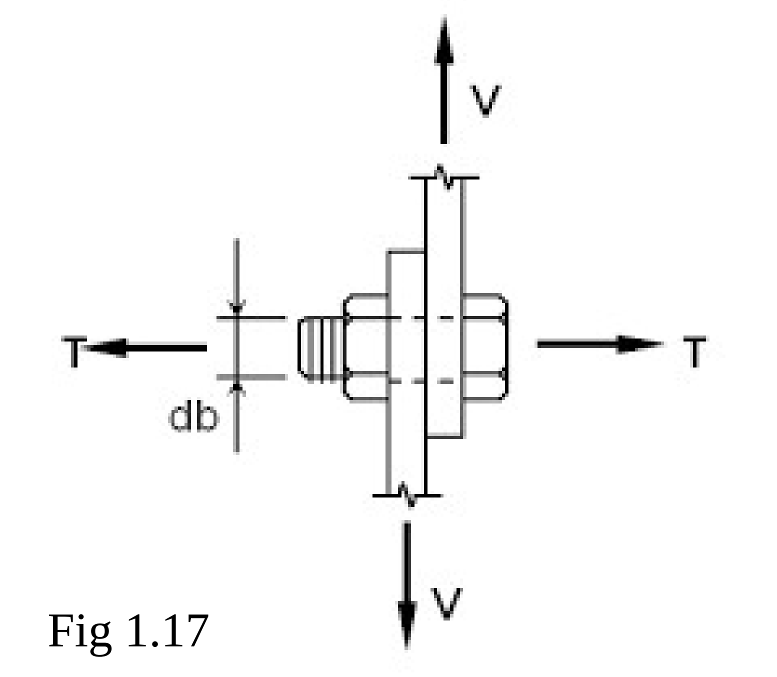


Fig 1.17

- Consider the two plates A and B, which are connected by a bolt CD (Fig.1.18).

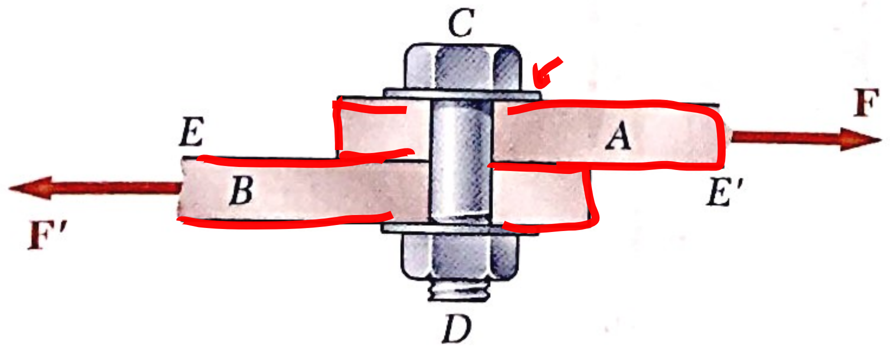


Fig. 1.18

- If the plates are subjected to tension forces of magnitude ***F***, stresses will develop in the section of bolt corresponding to the plane ***EE'***.
- Drawing the diagram of the bolt and of the portion located above the plane ***EE'*** (Fig.1.19), it is concluded that the shear P in the section is equal to ***F***.

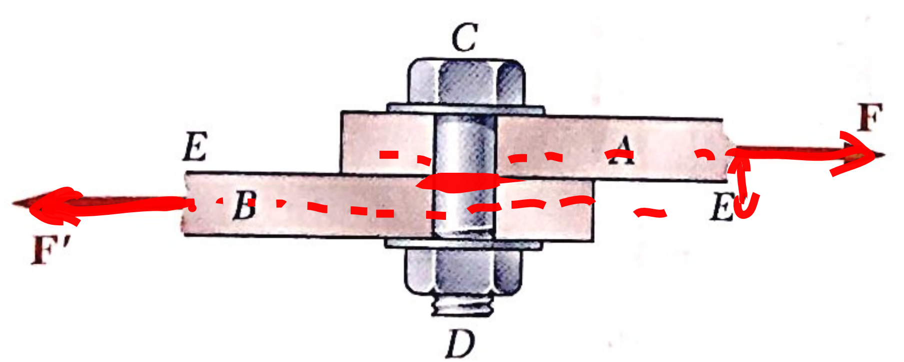


Fig. 1.18

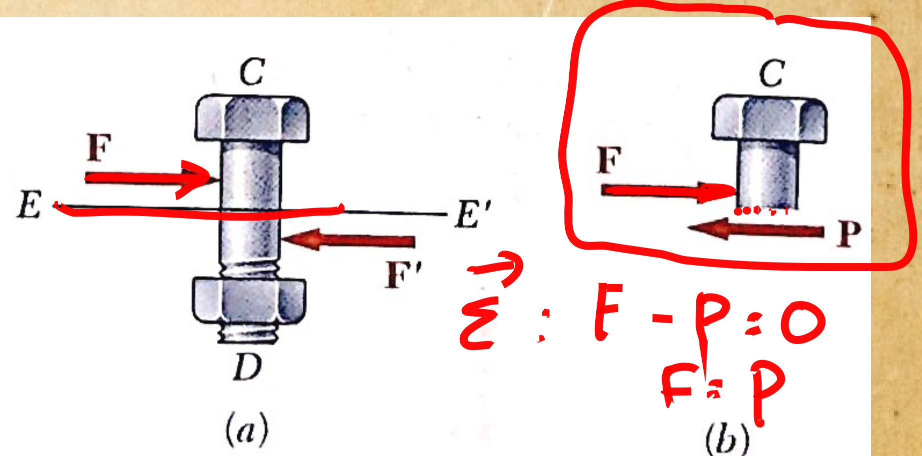
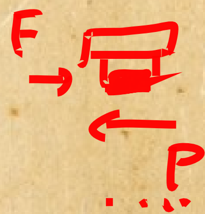


Fig. 1.19

$\tau_{Ave} = \frac{P}{A}$ (1.8)

- The average shearing stress in the section is obtained, according to formula (1.8), by dividing the shear ***P*=*F*** by the area A of the cross section.

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A} \qquad (1.9)$$



- The bolt we have just considered is said to be in ***single shear.***
- Different loading situations may arise, however.

- For example, if splice plates ***C*** and ***D*** are used to connect plates ***A*** and ***B*** (Fig.1.20), shear will take place in bolt ***HJ*** in each of the two planes ***KK'*** and ***LL'*** (and similarly in bolt ***EG***).
- The bolts are said to be in ***double shear***.

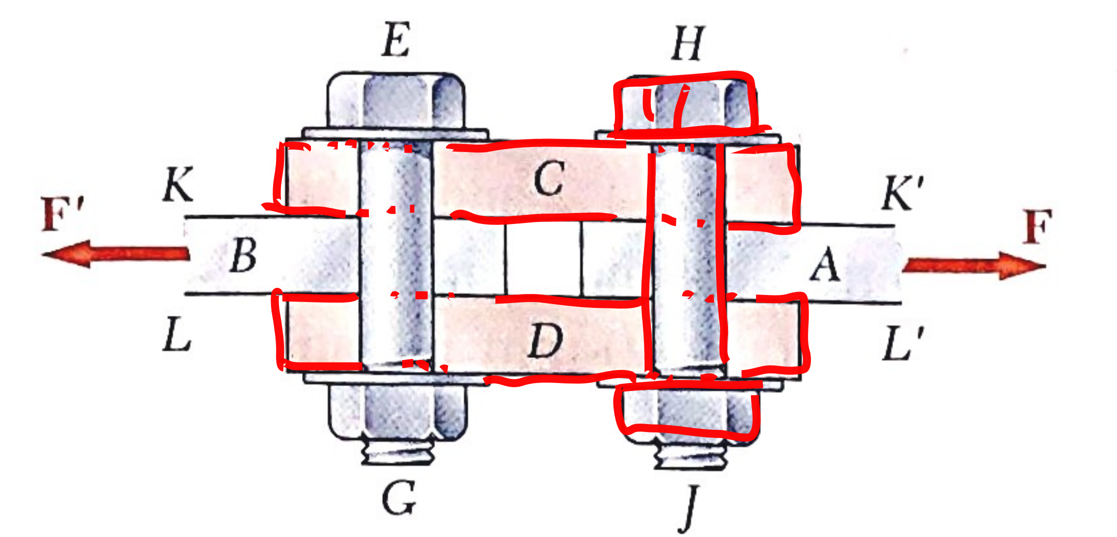


Fig. 1.20

- To determine the average shearing stress in each plane, it can be drawn <u>free-body diagrams</u> of bolt ***HJ*** and of the portion of bolt located between the two planes (Fig.1.21).

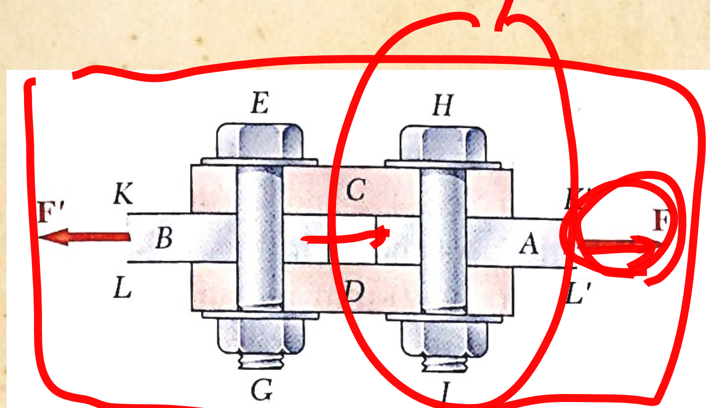


Fig. 1.20

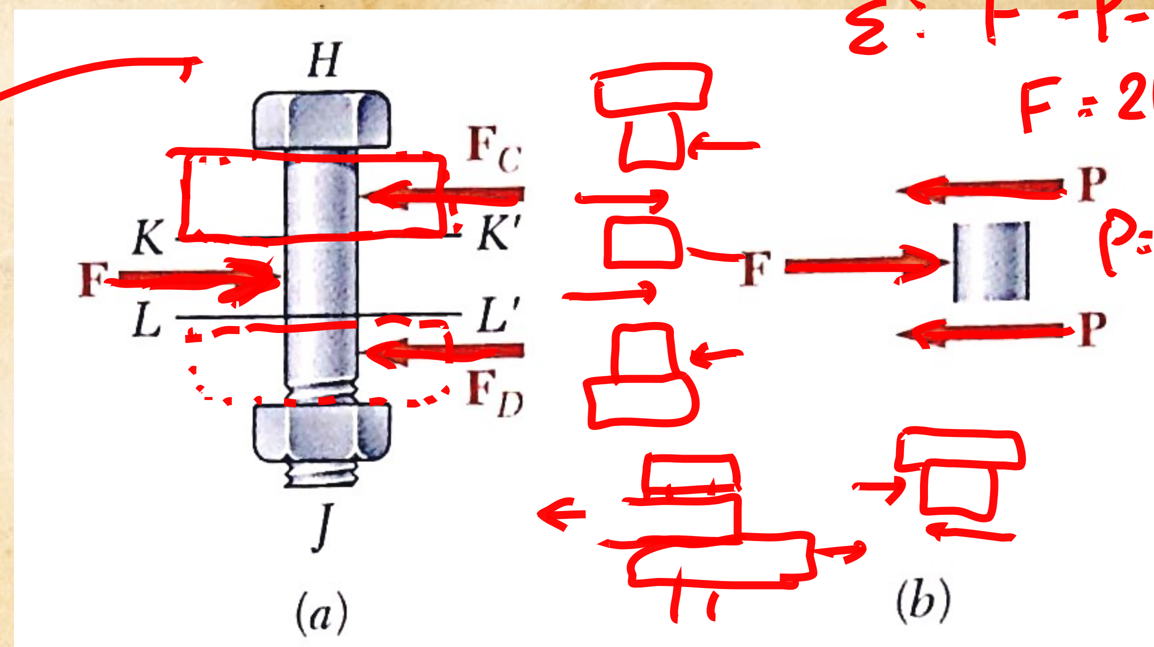


Fig. 1.21

$\overrightarrow{\Sigma}: F - P - P = 0$

$F = 2P$

$P = \frac{F}{2}$

- Observing that the shear ***P*** in each of the sections is ***P=F/2***, it is concluded that the average shearing stress is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A} \qquad (1.10)$$

Bearing stress in connections

- Bolts, pins, and rivets create stresses in the members they connect, along the ***bearing surface***, or surface of contact.
- For example, consider again the two plates ***A*** and ***B*** connected by a bolt ***CD*** that we have discussed in the preceding section (Fig.1.18).

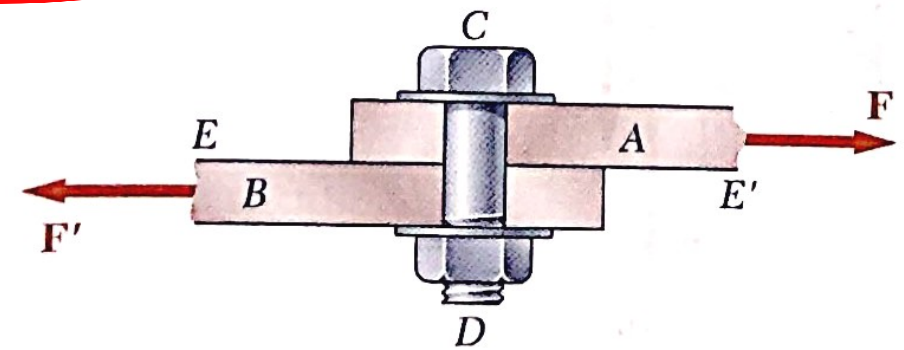


Fig. 1.18

- The bolt exerts on plate ***A*** a force ***P*** equal and opposite to the force ***F*** exerted by the plate on the bolt (Fig.1.22).

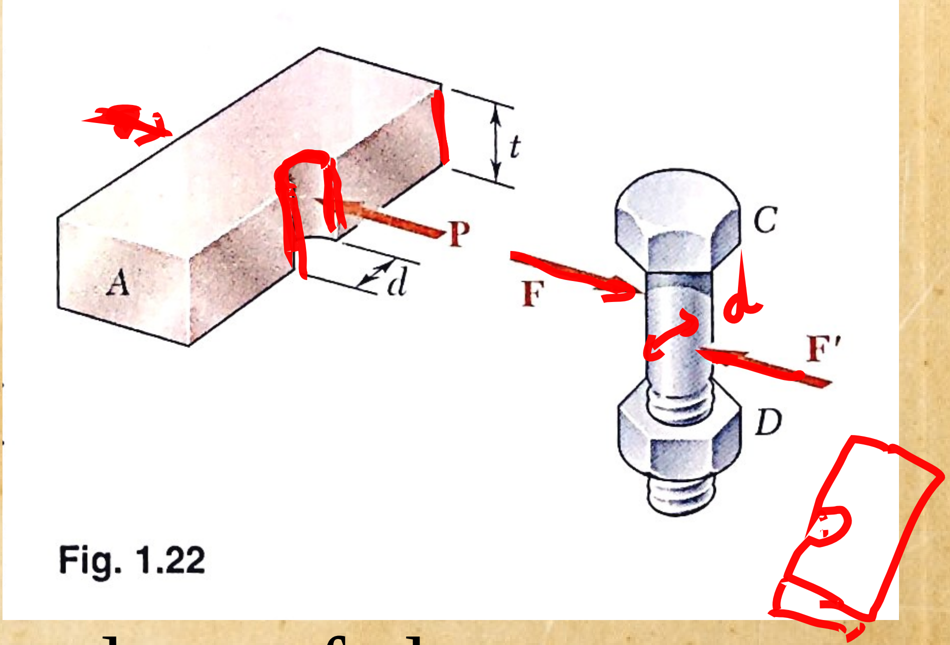


Fig. 1.22

- The force ***P*** represents the resultant of elementary forces distributed on the inside surface of a half-cylinder of diameter ***d*** and of length ***t*** equal to the thickness of the plate.

- Since the distribution of these forces – and of the corresponding stresses – is quite complicated, one uses in practice an average nominal value $\boldsymbol{\sigma_b}$ of the stress, called ***the bearing stress***, obtained by dividing the load ***P*** by the area of the rectangle representing the projection of the bolt on the plate section (Fig.1.23).

td

$\frac{P}{}$

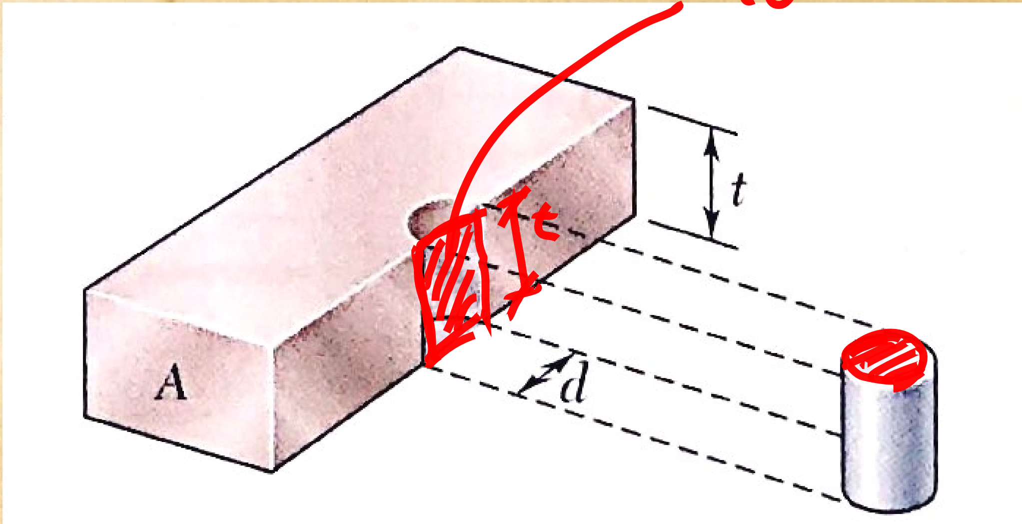


Fig. 1.23

- Since this area is equal to ***td***, where ***t*** is the plate thickness and d the diameter of the bolt, we can have

$$\sigma_b = \frac{P}{A} = \frac{P}{td} \tag{1.11}$$

Design

- The engineer's role is not limited to the analysis of existing structures and machines subjected to given loading conditions.
- Of even greater importance to the engineer is <u>*the design of new structures and machines*</u>, that is, the selection of appropriate components to perform a given task.

Example of Design using Fig.1.1

- Now let's assume that <u>*aluminum*</u> with an allowable stress $\boxed{\boldsymbol{\sigma_{all}=100MPa}}$ is to be used.
- Since the force in rod ***BC*** will still be

$$P=F_{BC}=50kN$$

under the given loading applied,

50kN

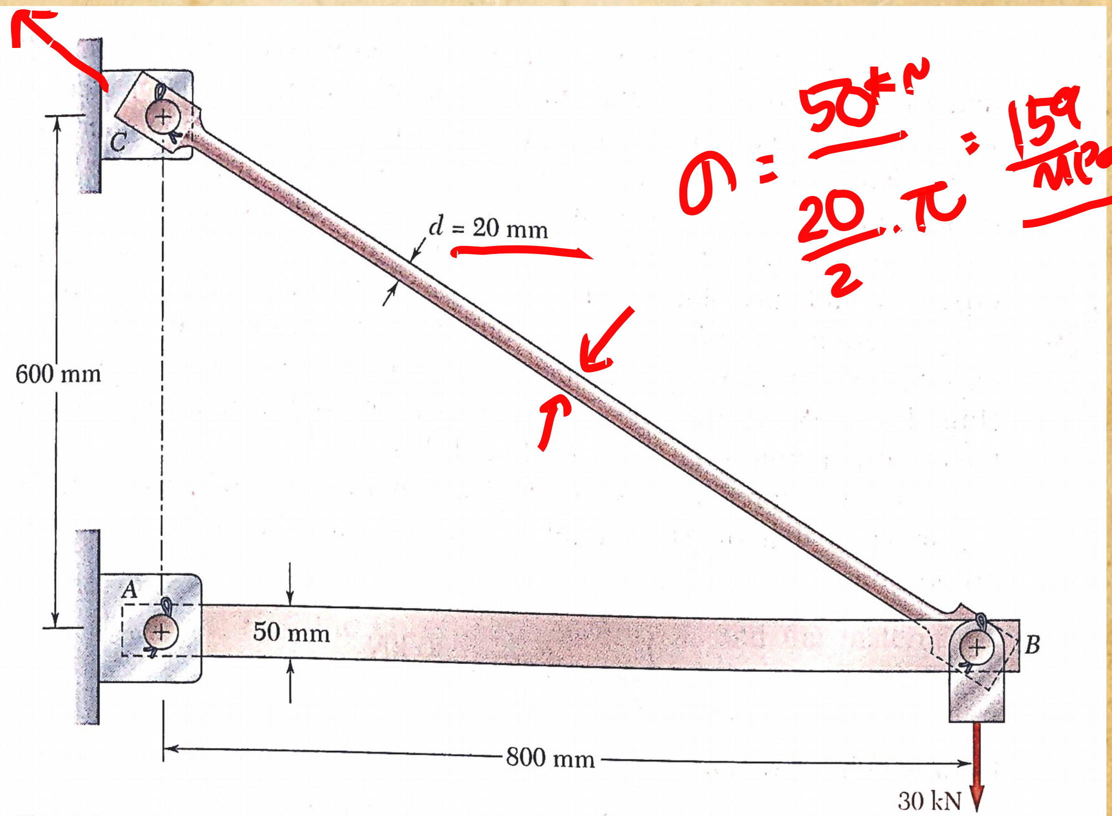


Fig. 1.1

$\sigma = \dfrac{50kN}{\frac{20}{2} . \pi} = 159\ MPa$

$\sigma = \frac{P}{A}$

- Since the value obtained for ***σ (=159 MPa)*** is bigger than the value ***σ_{all} (=100 Mpa)*** of the allowable stress in the aluminum used, it must be re-designed the ***Area of rod BC*** by the following equation;

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{\overset{50kN}{\boxed{P}}}{\sigma_{all}} = \frac{50 \times 10^3 \text{ N}}{100 \times 10^6 \text{ Pa}} = 500 \times 10^{-6} \text{ m}^2$$

Pa

$A = a^2$

σ_{all}

$P =$

- Since ***A=πr²***,

$A = \frac{P}{\sigma_{all}} = \_\_\_$

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{500 \times 10^{-6}\ \mathrm{m}^2}{\pi}} = 12.62 \times 10^{-3}\ \mathrm{m} = 12.62\ \mathrm{mm}$$

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$$d = 2r = \boxed{25.2\ \mathrm{mm}}$$

- It is concluded that an *aluminum rod* ***26 mm or more in diameter*** will be adequate.

Mini Quiz 5

~~Question 3~~

Two solid cylindrical rods ***AB*** and ***BC*** are welded together at ***B*** and loaded as shown.

Knowing that the average normal stress must not exceed $\sigma_{all} =$ ***175 MPa*** in rod ***AB*** and ***150 MPa*** in rod ***BC***, determine the smallest allowable values of Q1 $\boxed{d_1}$ and $\boxed{d_2}$. Q2