Introduction into design engineering week 10

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Stress under general loading conditions; components of stress

- The examples of the previous sections were limited to members under axial loading and connections under transverse loading.

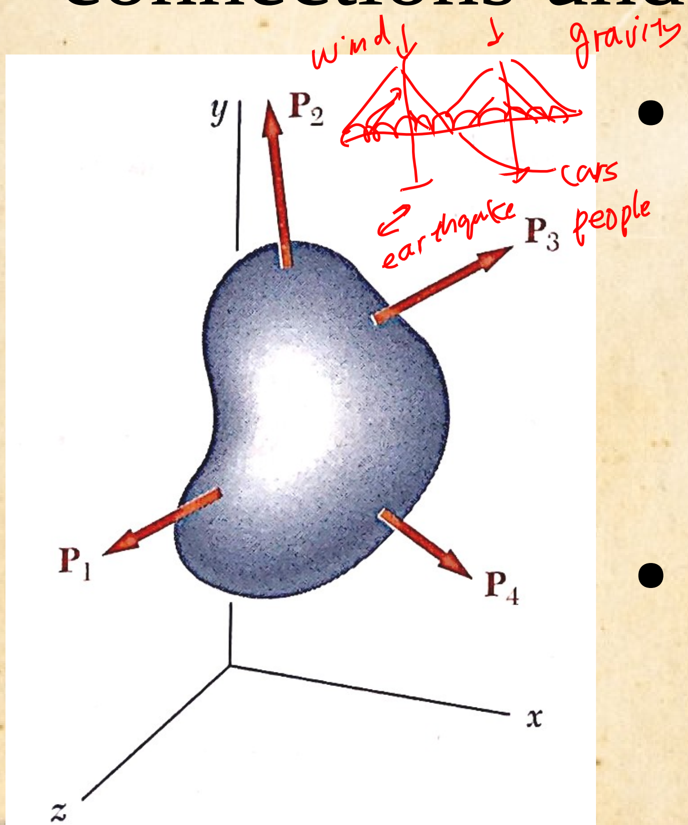


Fig. 1.32

- Most structural members and machine components are under more involved loading conditions.
- Consider a body subjected to several loads P_1, P_2, etc. (Fig.1.32).

- To understand the stress condition created by these loads at some point ***Q*** within the body, it shall be first passed a section through ***Q***, using a plane parallel to the *yz* plane.

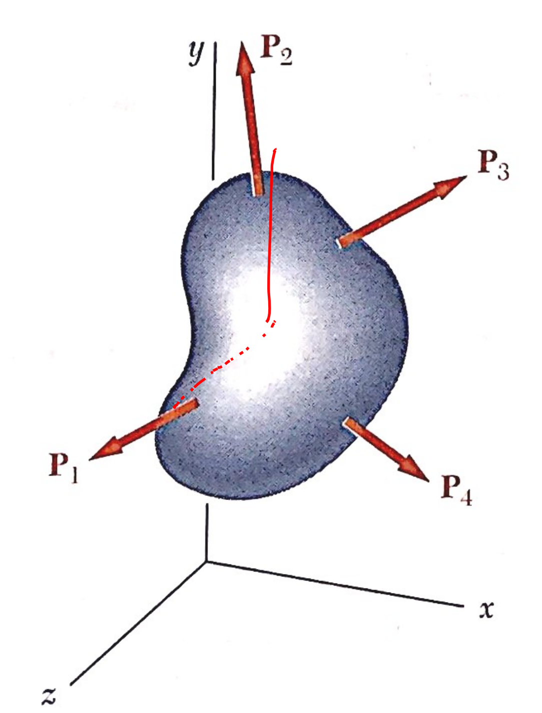
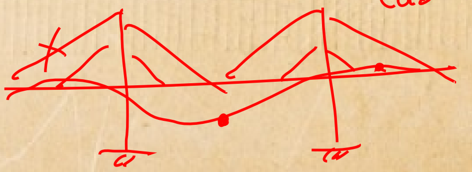


Fig. 1.32

- The portion of the body to the left of the section is subjected to some of the original loads, and to normal and shearing forces distributed over the section.

斜張橋

cable-stayed bridge



- It shall be denoted by $\boldsymbol{\Delta F^x}$ and $\boldsymbol{\Delta V^x}$, respectively, the normal and the shearing forces acting on a small area $\boldsymbol{\Delta A}$ surrounding point $\boldsymbol{Q}$ (Fig.1.33a).

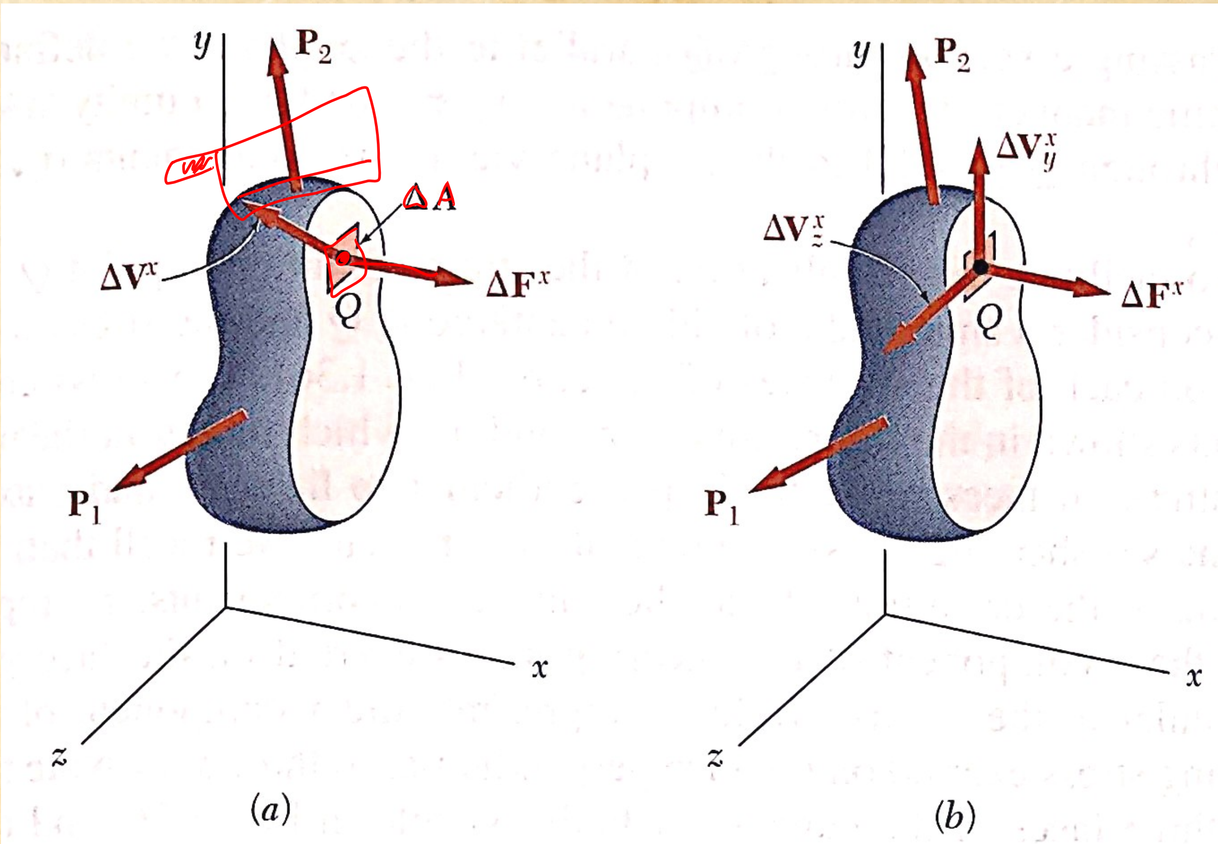


Fig. 1.33

- Note that the superscript *x* is used to indicate that the force ΔF^x and ΔV^x *act* on a surface perpendicular to the x axis.
- While the normal force ΔF^x has a well-defined direction, the shearing force ΔV^x may have any direction in the plane of the section.

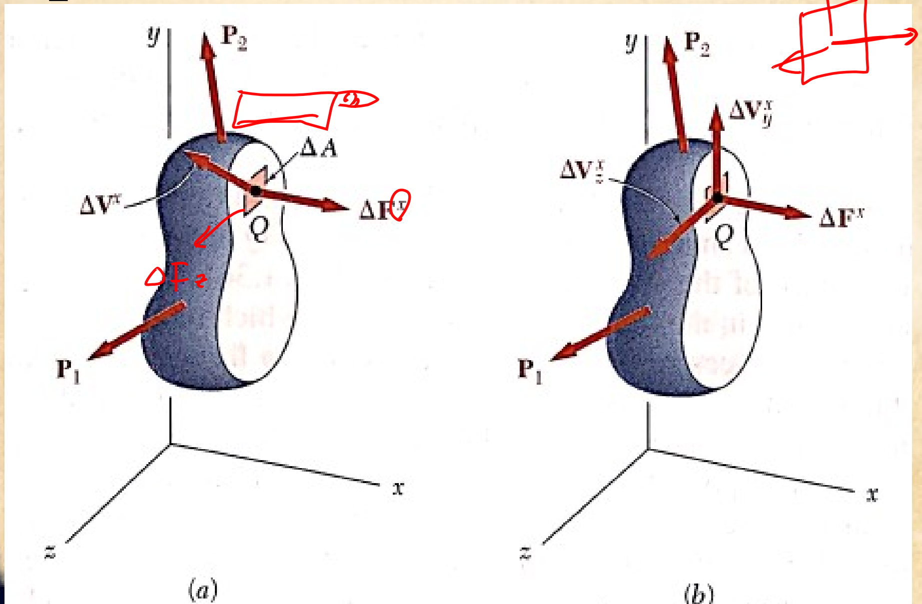


Fig. 1.33

- Therefore, it should be resolved ΔV^x into two component forces, ΔV^x_y and ΔV^x_z in directions parallel to the *y* and *z* axes, respectively (Fig. 1.33 b).

- Dividing now the magnitude of each force by the area ***ΔA***, and letting ***ΔA*** approach zero, it is defined the three stress components shown in Fig.1.34:

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A} \tag{1.18}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$

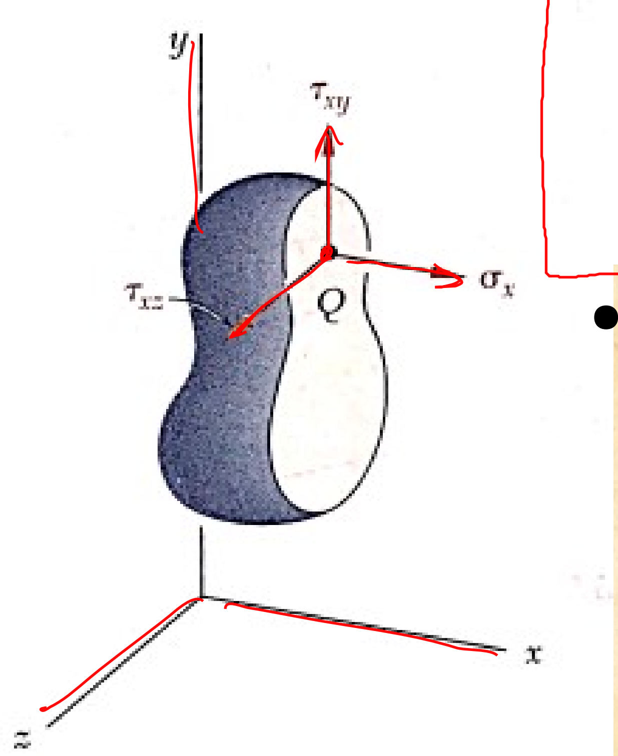


Fig. 1.34

- It is noted that the first subscript in σ_x, τ_{xy}, and τ_{xz} is used to indicate that the stresses under consideration are exerted *on a surface perpendicular to the x axis*.

- The second subscript in τ_{xy} and τ_{xz} identifies *the direction of the component.*
- The normal stress σ_x is positive if the corresponding arrow points in the positive *x* direction, i.e., if the body is in tension, and negative otherwise.
- Similarly, the shearing stress components τ_{xy} and τ_{xz} are positive if the corresponding arrows point, respectively, in the positive *y* and *z* directions.

- The above analysis may also be carried out by considering the portion of body located to the right of the vertical plane through *Q* (Fig.1.35).

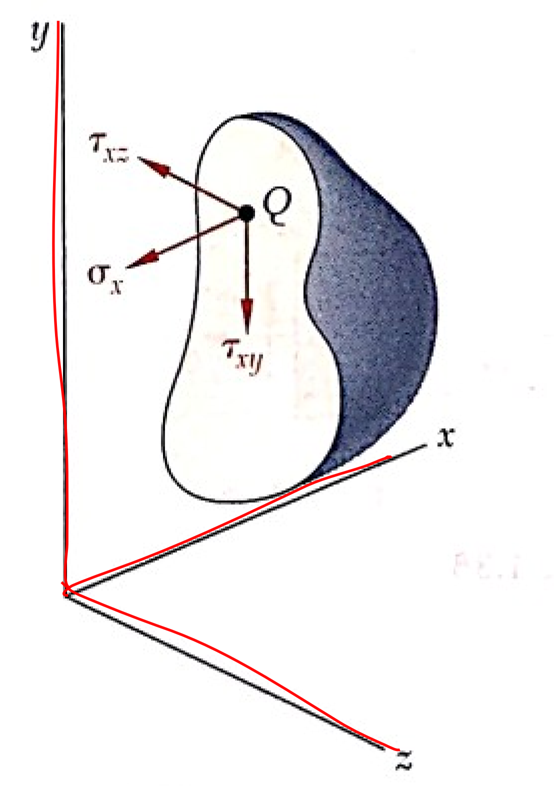


Fig. 1.35

- The same magnitudes, but opposite directions, are obtained for the normal and shearing forces $\boldsymbol{\Delta F^x}$, $\boldsymbol{\Delta V^x_y}$ and $\boldsymbol{\Delta V^x_z}$.

- Therefore, the same values are also obtained for the corresponding stress components, but since the section in Fig.1.35 now faces the *negative x axis*, a positive sign for σ_x will indicate that the corresponding arrow points *in the negative x direction*.

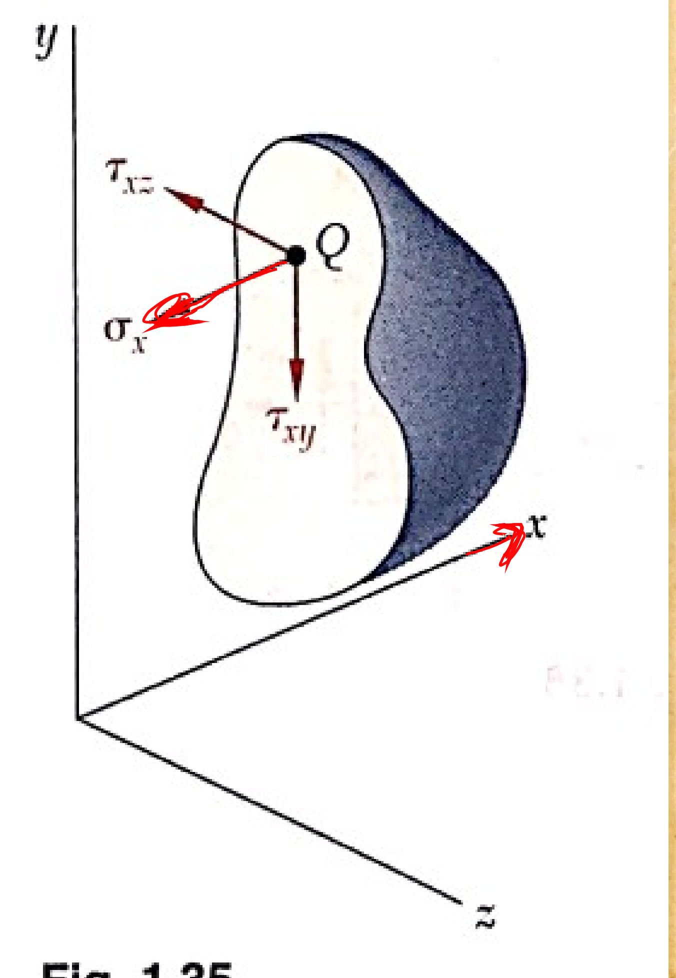


Fig. 1.35

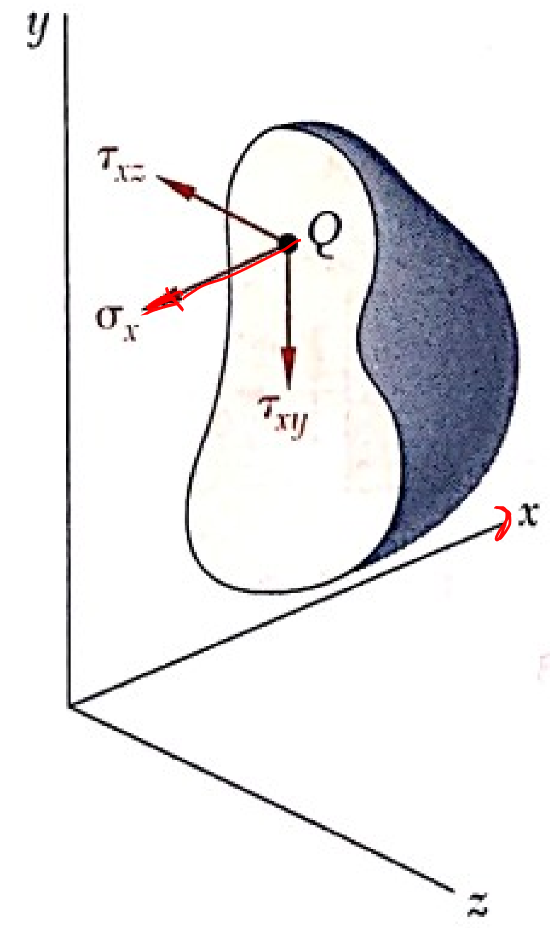


Fig. 1.35

- Similarly, positive signs for τ_{xy} and τ_{xz} will indicate that the corresponding arrows point, respectively, in the negative *y* and *z* directions, as shown in Fig.1.35.

- Passing a section through Q parallel to the zx plane, it is defined in the same manner the stress components, σ_y, τ_{yz}, and τ_{yx}.
- Finally, a section through Q parallel to the xy plane yields the components σ_z, τ_{zx}, and τ_{zy}.

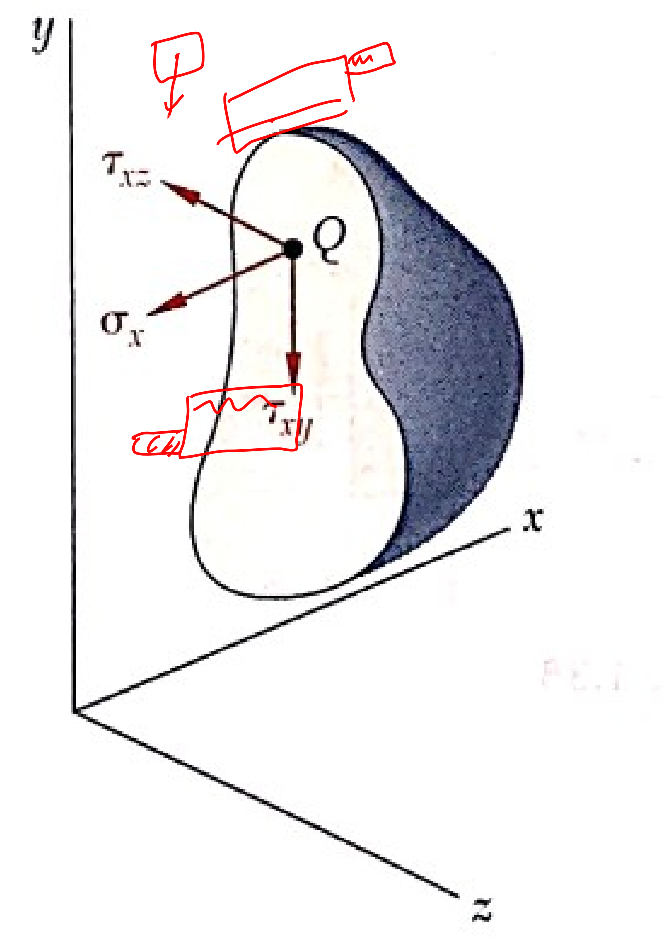
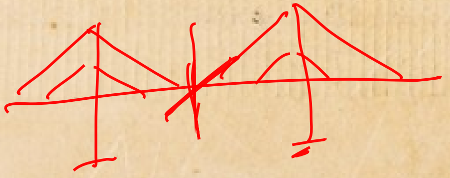


Fig. 1.35



- To facilitate the visualization of the stress condition at point *Q*, it shall be considered a small cube of side a centered at *Q* and the stresses exerted on each of the six faces of the cube (Fig.1.36).

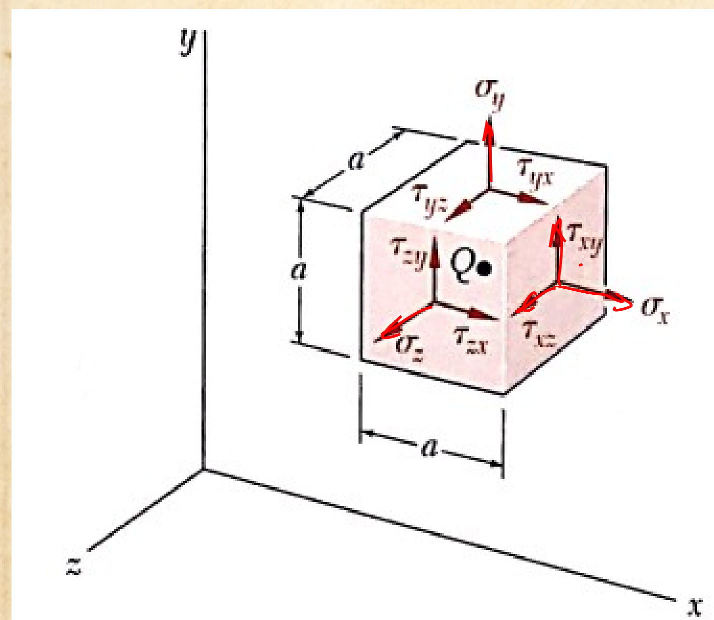


Fig. 1.36

- The stress components shown in the figure are σ_x, σ_y, and σ_z, which represent the normal stress on faces respectively perpendicular to the *x*, *y*, *and z* axes, and the six shearing stress components τ_{xy}, τ_{xz}, *etc*.

- According to the definition of the shearing stress components, τ_{xy} represents the y component of the shearing stress exerted on the face perpendicular to the x axis, while τ_{yx} represents the x component of the shearing stress exerted on the face perpendicular to the y axis.

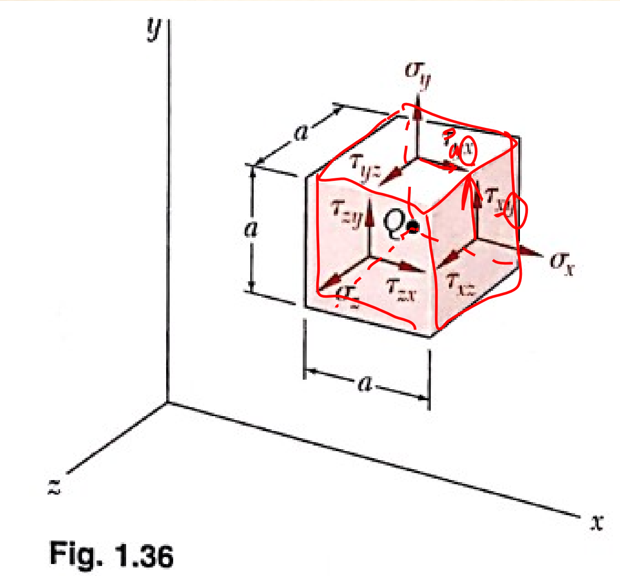


Fig. 1.36

- Note that only three faces of the cube are actually visible in Fig.1.36, and that equal and opposite stress components act on the hidden faces.

- While the stresses acting on the faces of the cube differ slightly from the stresses at *Q*, the error involved is small and vanishes as side a of the cube approaches zero.

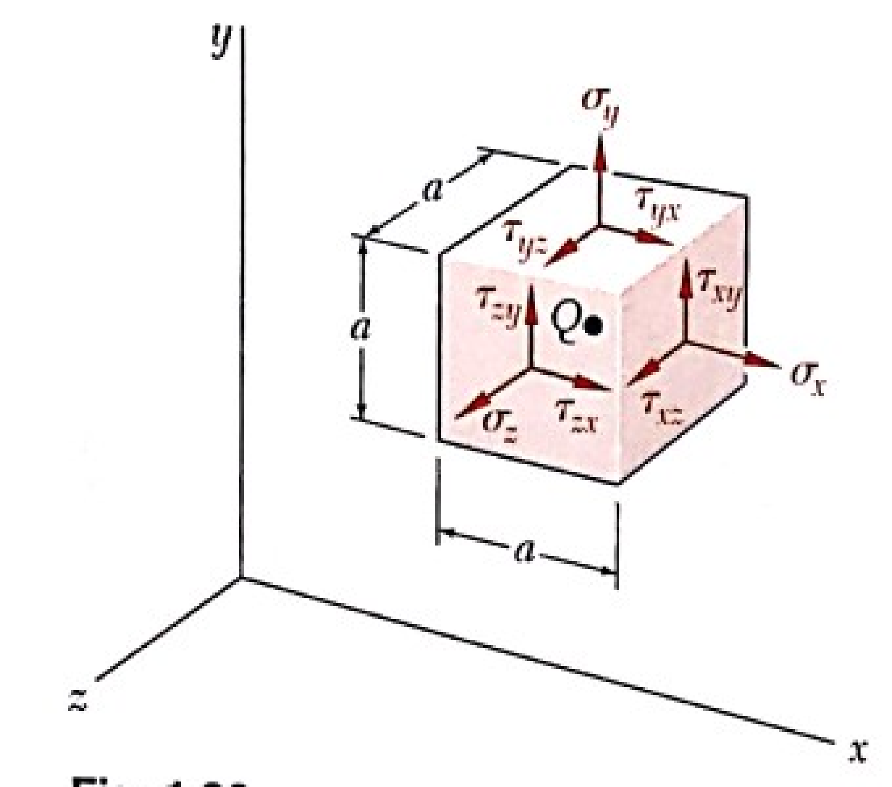


Fig. 1.36

$\sigma = \frac{P}{A}$ $\sigma \cdot \Delta A = P$

$\tau = \frac{P}{\Delta A} \Rightarrow \tau \Delta A = P$

- Important relations among the shearing stress components will now be derived.
- Let's consider the free-body diagram of the small cube centered at point *Q* (Fig.1.37).

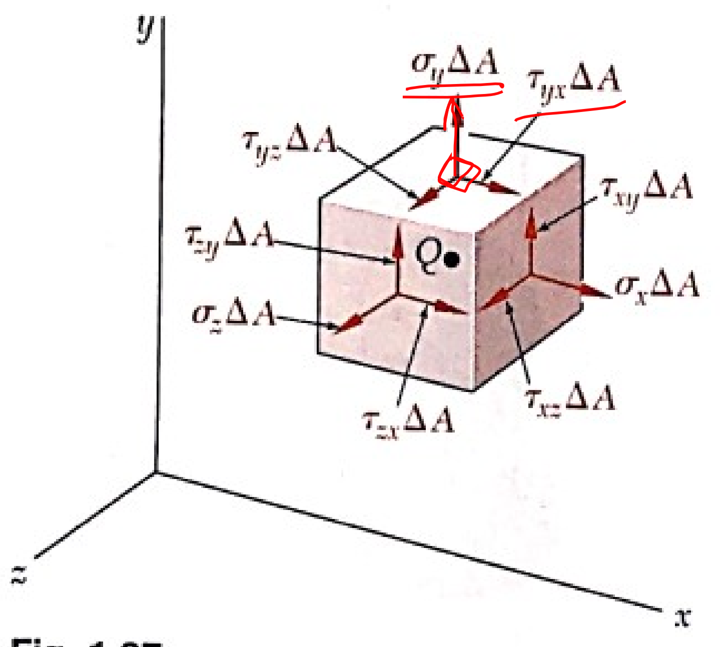


Fig. 1.37

- The normal and shearing forces acting on the various faces of the cube are obtained by multiplying the corresponding stress components by the area *ΔA* of each face.

- At first, these three equilibrium equations can be written;

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \qquad (1.19)$$

- Since forces equal and opposite to the forces actually shown in Fig.1.37 are acting on the hidden faces of the cube, it is clear that Eqs. (1.19) are satisfied.

$\tau_{yx} = \tau_{xy}$ $\tau_{yz} = \tau_{zy}$
$\tau_{xz} = \tau_{zx}$

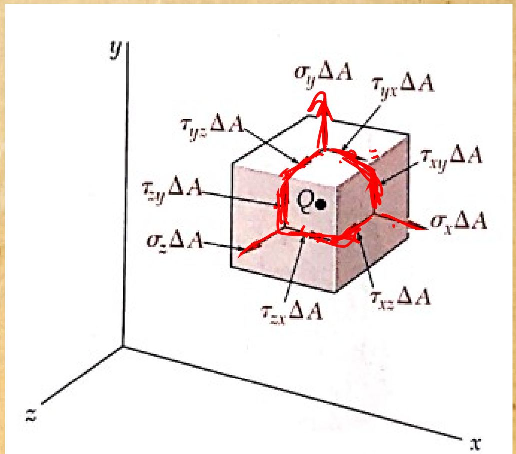


Fig. 1.37

- Considering now the moments of the forces about axes Q'_x, Q'_y, and Q'_z drawn from Q in directions respectively parallel to the x, y, and z axes, three additional equations are written:

$$\Sigma M_{x'} = 0 \qquad \Sigma M_{y'} = 0 \qquad \Sigma M_{z'} = 0 \qquad (1.20)$$

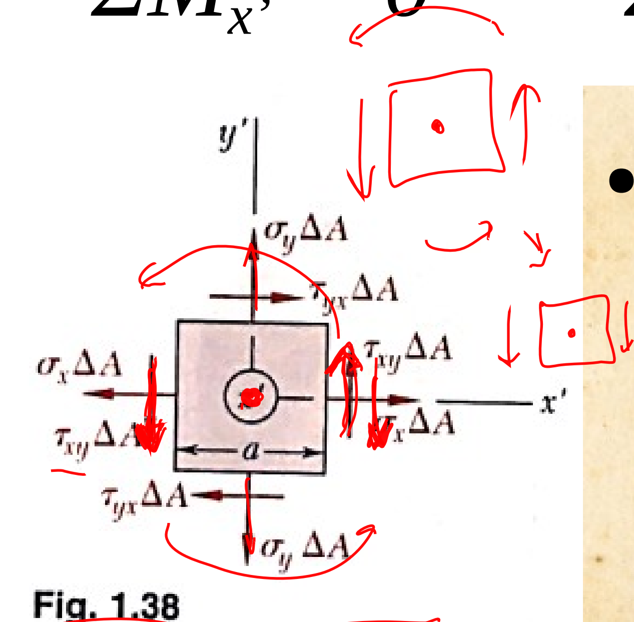


Fig. 1.38

- Using a projection on the x’ y’ plane (Fig.1.38), it is noted that the only forces with moments about the z axis different from zero are the shearing forces.

- These forces form two couples, one of counterclockwise (positive) moment $(\tau_{xy}\Delta A)a$, the other of clockwise (negative) moment $-(\tau_{xy}\Delta A)a$.
- The last of the three Eqs.(1.20) yields, therefore,

$$+\circlearrowleft \Sigma M_z = 0 : \qquad (\tau_{xy}\Delta A)a - (\tau_{xy}\Delta A)a = 0$$

- From which it is concluded that

$$\tau_{xy} = \tau_{yx} \qquad (1.21)$$

- The relation obtained shows that the *y* component of the shearing stress exerted on a face perpendicular to the *x axis* is equal to the *x* component of the shearing stress exerted on a face perpendicular to the *y axis*.

- From the remaining two equations (1.20), it is derived in a similar manner the relations

$$\tau_{yz} = \tau_{zy} \qquad \tau_{zx} = \tau_{xz} \qquad (1.22)$$

- It is concluded from Eqs.(1.21) and (1.22) that only six stress components are required to define the condition of stress at a given point *Q*, instead of nine as originally assumed.
- These six components are σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx}.

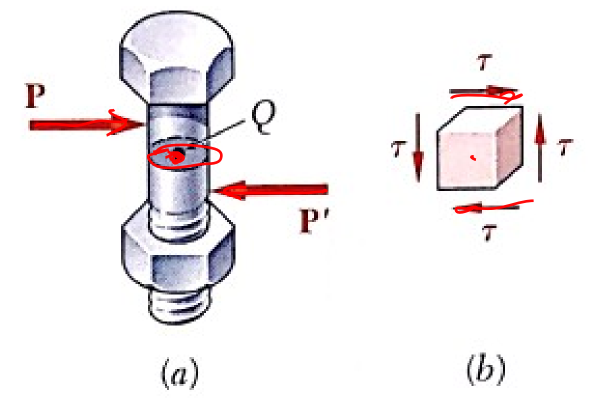
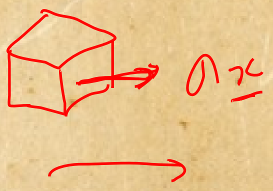


Fig. 1.39

- It is noted that, at a given point, *shear cannot take place in one plane only*; an equal shearing stress must be exerted on another plane perpendicular to the first one.

- For example, considering the bolt and a small cube at the center *Q* of the bolt (Fig.1.39a), is is found that shearing stresses of equal magnitude must be exerted on the two horizontal faces of the cube and on the two face that are perpendicular to the forces ***P*** and ***P'*** (Fig.1.39b).

- Before concluding our discussion of stress components, let’s consider again the case of a member under axial loading.



- If it is considered a small cube with faces respectively parallel to the faces of the member and recall the results obtained in Sec.1.11, it is found that the conditions of stresses are normal stresses σ_x exerted on the faces of the cube which are perpendicular to the x axis.

- However, if the small cube is rotated by 45° about the z axis so that its new orientation matches the orientation of the sections considered in Fig.1.31c and d, it is concluded that normal and shearing stresses of equal magnitude are exerted on four faces of the cube (Fig. 1.40b).

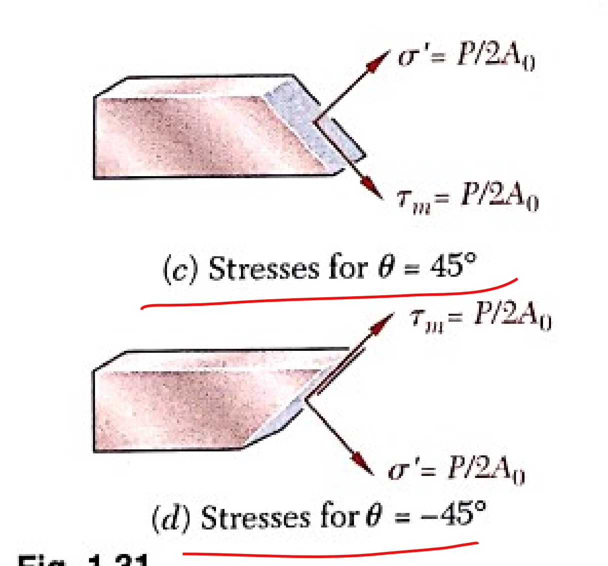


Fig. 1.31

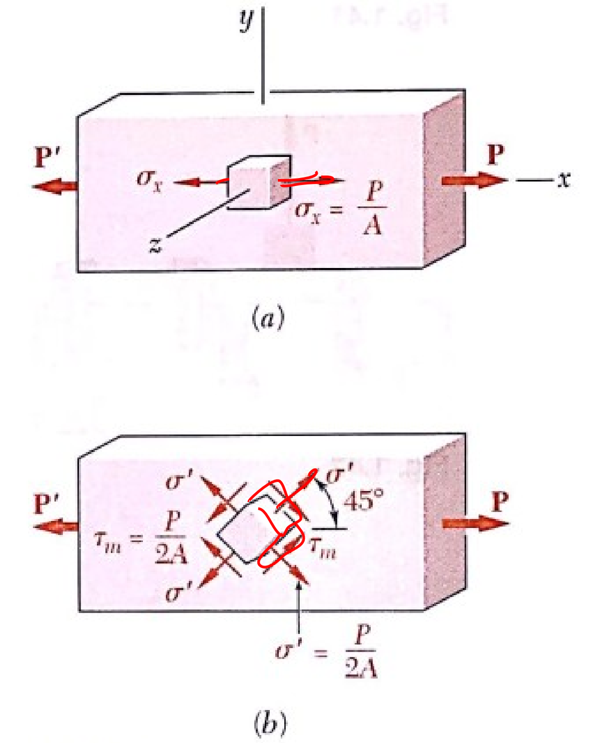


Fig. 1.40

- However, if the small cube is rotated by 45° about the z axis so that its new orientation matches the orientation of the sections

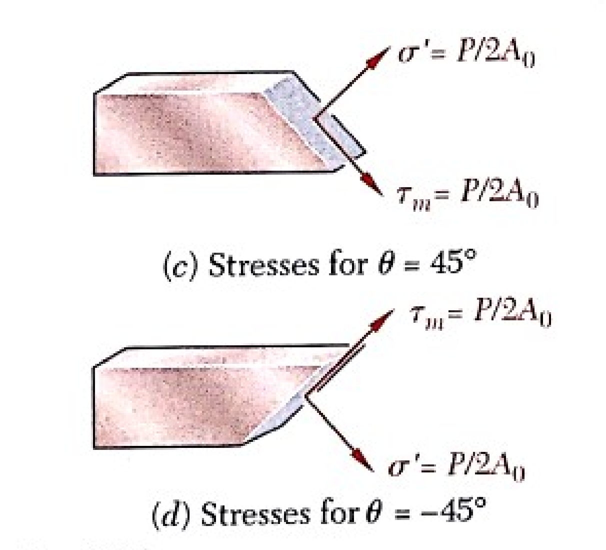


Fig. 1.31

considered in Fig.1.31c and d, it is concluded that normal and shearing stresses of equal magnitude are exerted on four faces of the cube (Fig. 1.40b).

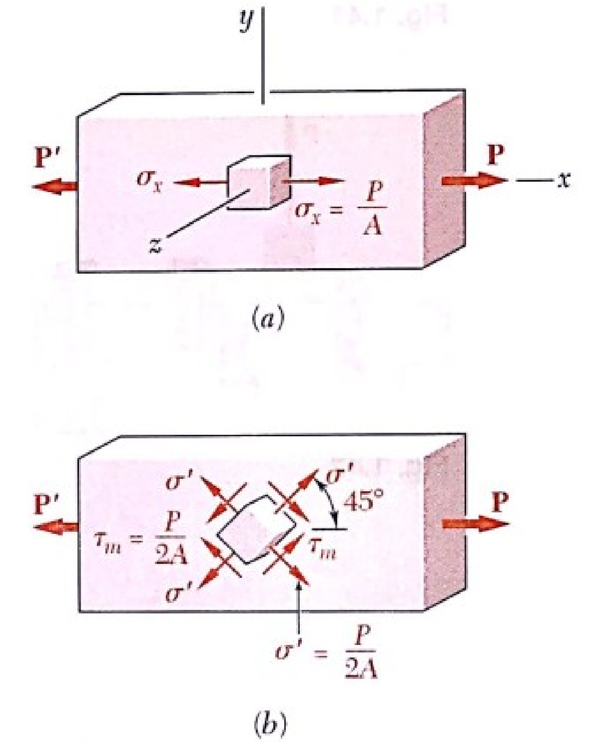
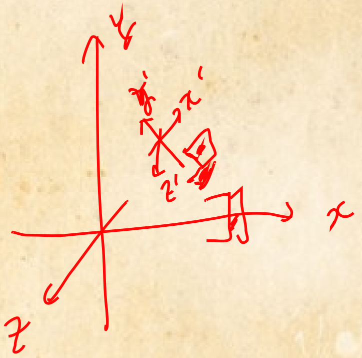
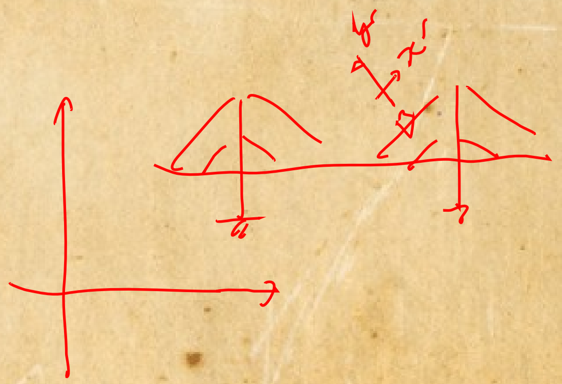


Fig. 1.40

- Thus, it should be observed that the same loading condition may lead to different interpretations of the stress situation at a given point, depending upon the orientation of the element considered.





Mini quiz week 10

- Please search and explain 10 keywords from today’s class in English.