Introduction into design engineering week 4

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Axial loading; normal stress

- The force already been indicated in the previous session, the forced F_{BC} and F'_{BC} acting on its ends B and C (see in Fig.1.15) are directed along the axis of the rod.

- It is said that the rod is ***under axial loading***.

axial force

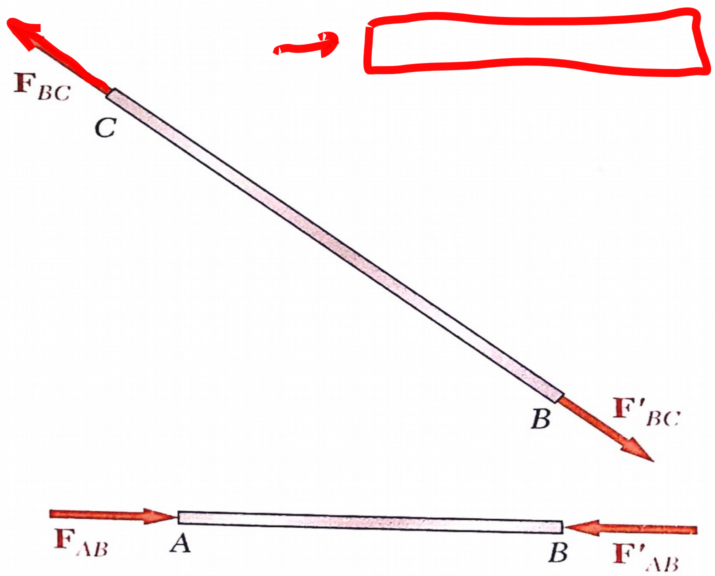
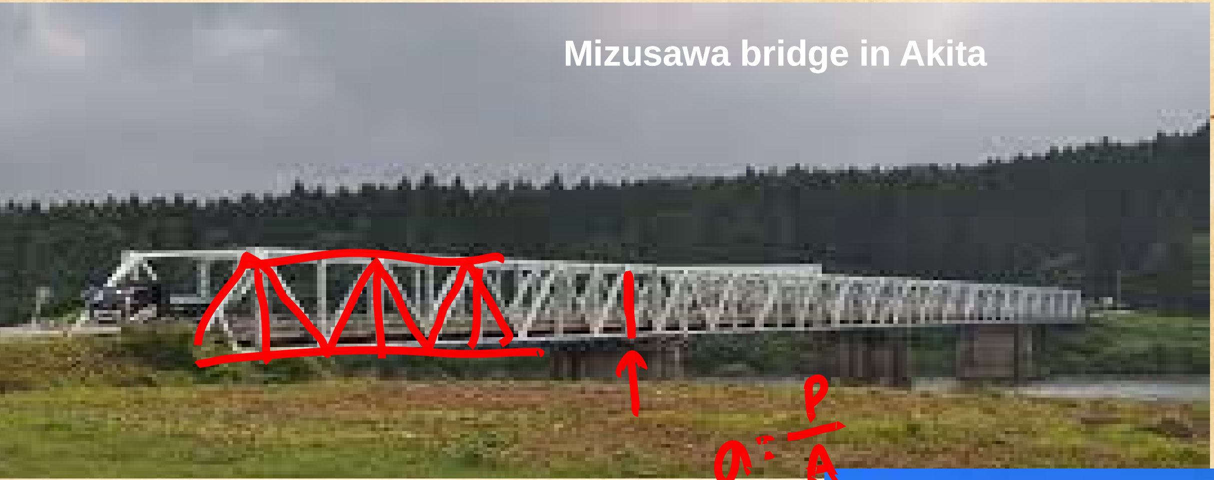


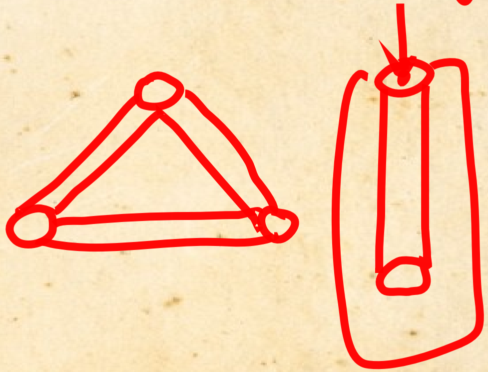
Fig. 1.5

An example of axial loads



These bridges are called 'truss bridge' which consists of two-force members that may be in tension or in compression.

$\sigma = \frac{P}{A}$



axial load

→ normal stress

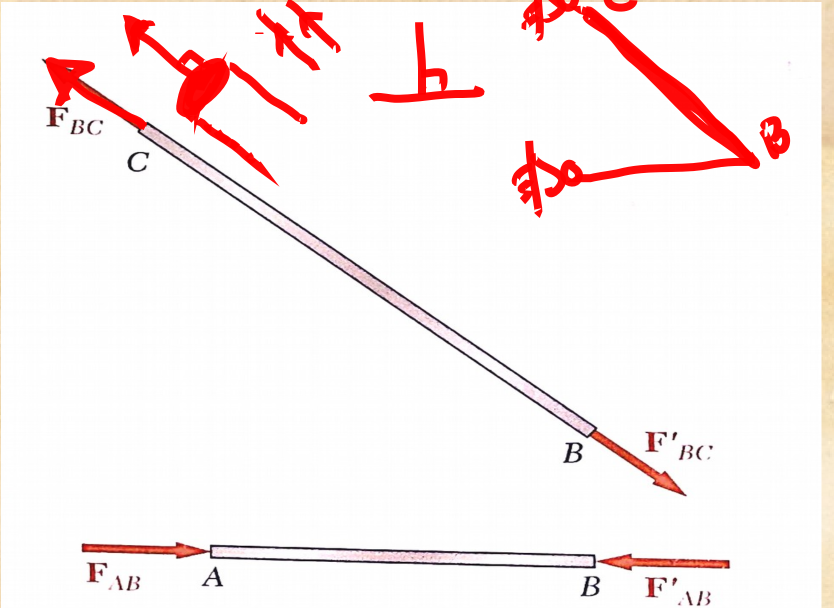


Fig. 1.5

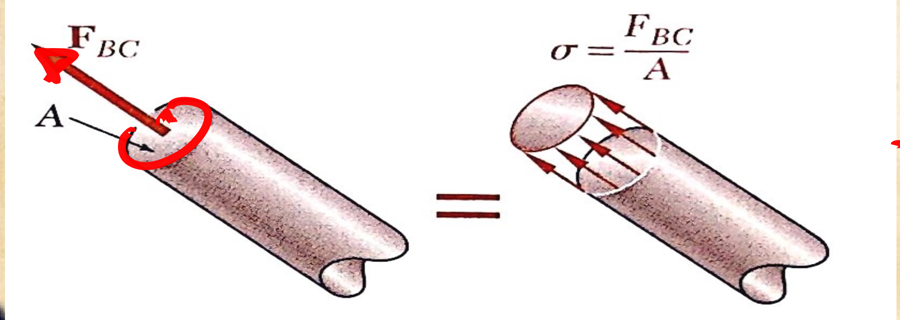


Fig. 1.7

- Let's think about rod BC of Fig.1.5,it should be reminded that the section parred through the rod to determine the internal force in the rod and the corresponding stress was perpendicular to the axis of the rod; the internal force was therefore normal to the plane of the section (see in Fig,1.7) and the corresponding stress is described as ***a normal stress***.

196kN

$\sigma = 25.0$ MPa

100

- Thus formula (1.5) $$\boxed{\sigma = \frac{P}{A}} \qquad (1.5)$$

  gives us the normal stress in a member under axial loading.

- It is also noted that, in formula (1.5), σ is obtained by dividing the magnitude P of the resultant of the internal forces distributed over the cross section by the area A of the cross section; it represents, therefore, the ***average value*** of the stress over the cross section, rather than the stress at a specific point of the cross

$\sigma = \frac{P}{A}$

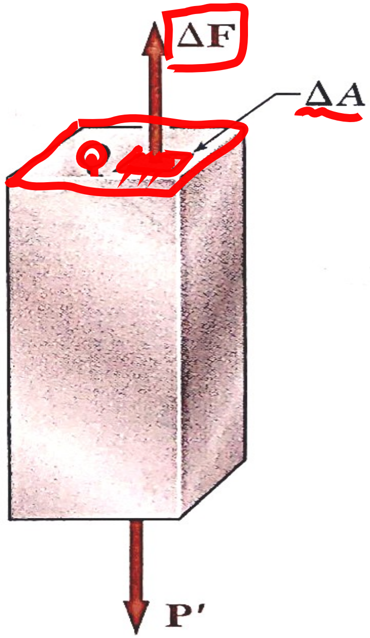


Fig. 1.10

- To define the stress at a given point Q of the cross section, we should consider a small area Δ***A*** (Fig.1.10).
- Dividing the magnitude of Δ***F*** by Δ***A*,** we obtain the average value of the stress over Δ***A***.
- Letting ΔA approach zero, we obtain

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \qquad (1.6)$$

- In general, the value obtained for the stress σ at a given point Q of the section is different from the value of the average stress given by formula (1.5), and σ is found to vary across the section.

$$\sigma = \frac{P}{A} \tag{1.5}$$

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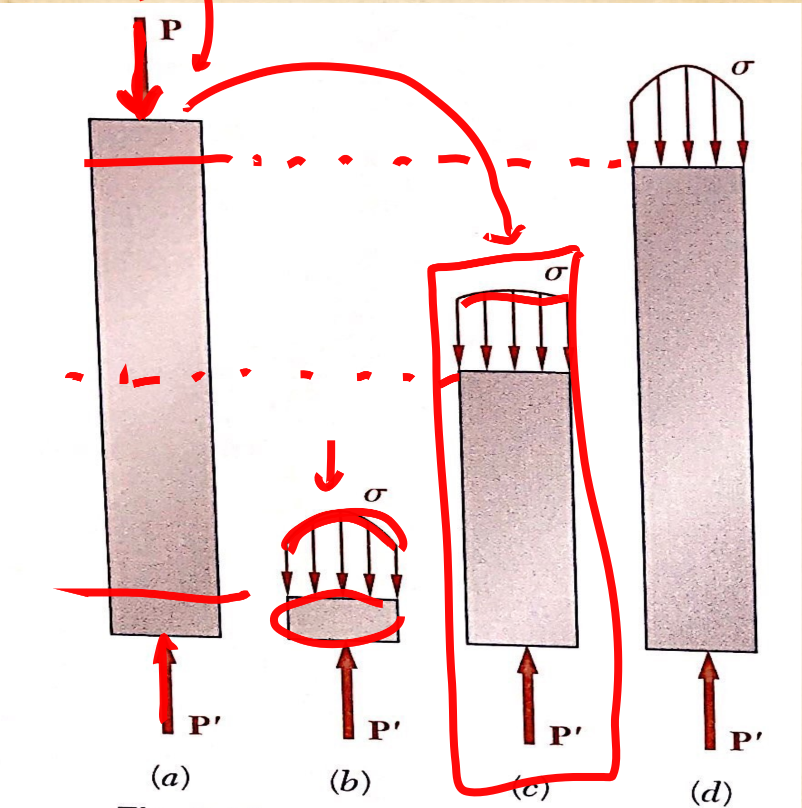

Fig. 1.11

- In a slender rod subjected to equal and opposite concentrated load **P** ad **P’** (Fig.1.11*a*), this variation is small in a section away from the points of application of the concentrated loads (Fig. 1.11*c*), but it is quite noticeable in the neighborhood of these points (Fig 1.11*b* and *d*).

and

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

- It follows from Eq.(1.6) that the magnitude of the resultant of the distributed internal forces is

$$\int dF = \int_A \sigma \, dA$$

- But the conditions of equilibrium of each of the portions of rod shown in Fig.1.11 require that this magnitude be equal to the magnitude P of the concentrated loads.

$\overrightarrow{\Sigma}$

$\uparrow\Sigma$

$\circlearrowleft\Sigma$

- Thus, it should be shown as

$$P = \int dF = \int_A \sigma \, dA \qquad (1.7)$$

Which means that the volume under each of the stress surfaces in Fig.1.11 must be equal to the magnitude P of the loads.

- This, however, is the only information that we can derive from our knowledge of statics, regarding the distribution of stresses in any given section is *statically indeterminate*.

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- In practice, it will be assumed that the distribution of normal stresses in an axially loaded member is uniform, except in immediate vicinity of the points of application of the loads.
- The value σ of the stress is then equal to σ_{ave} and can be obtained from formula (1.5).

$$\sigma = \frac{P}{A} \tag{1.5}$$

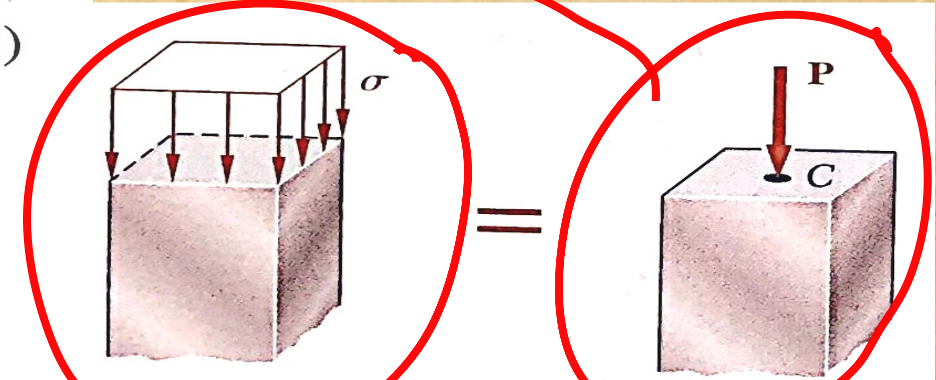

Fig. 1.12

- However, it should be realized that, when it is assumed that a uniform distribution of stresses in the section, i.e., when it is assumed that the internal forces are uniformly distributed across the section, it follows from elementary statics that the resultant ***P*** of the internal forces must be applied at the centroid *C* of the section (Fig.1.12).

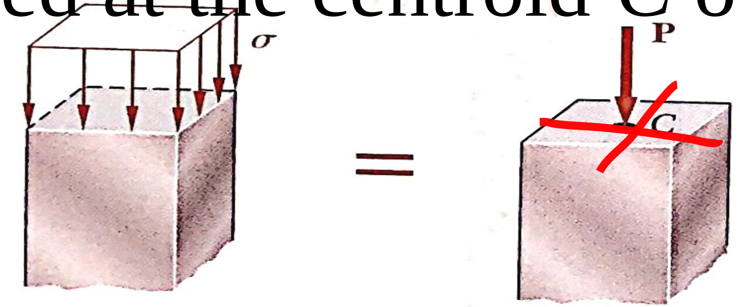

Fig. 1.12

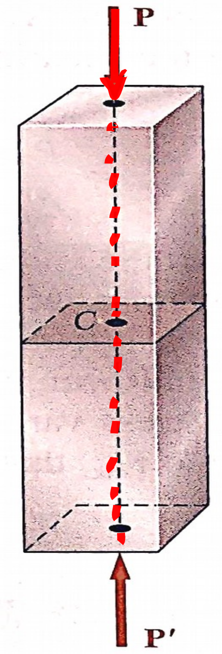

Fig. 1.13

- This means that *a uniform distribution of stress is possible only if the line of action of the concentrated loads P and P' passes through the centroid of the section considered* (Fig.1.13).
- This type of loading is called centric loading and will be assumed to take place in all straight two-force members found in trusses and pin-connected structures, such as the one considered in Fig.1.1.

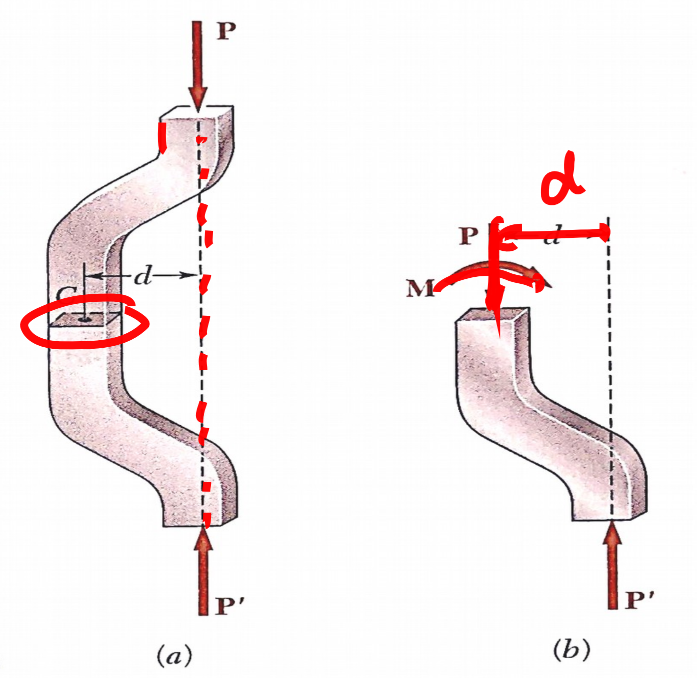

Fig. 1.14

- However, if a two-force member is loaded axially, but *eccentrically* as shown in Fig.1.14*a*, we find from the conditions of equilibrium of the portion of member shown in Fig.1.14*b* that the internal forces in a given section must be equivalent to force ***P*** applied at the centroid of the section and a couple ***M*** of moment ***M*=*Pd***.

- The distribution of forces – and, thus, the corresponding distribution of stresses – *cannot be uniform*.

distribution of stresses

- Nor can the distribution of stresses by symmetric as shown in Fig.1.11.

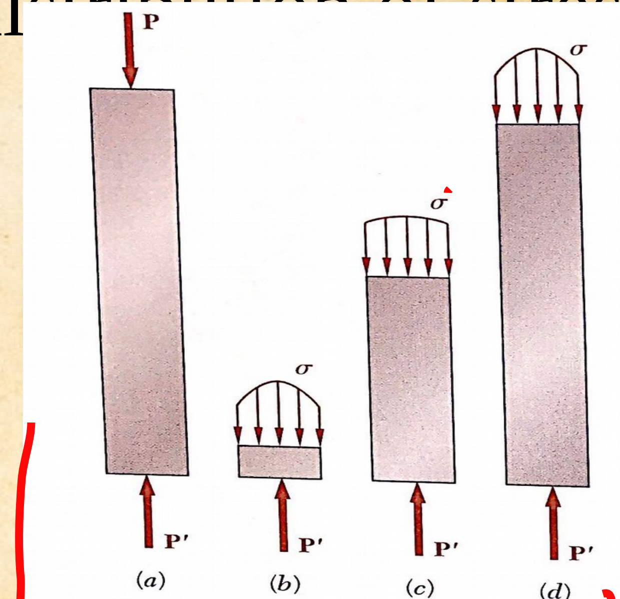

Fig. 1.11

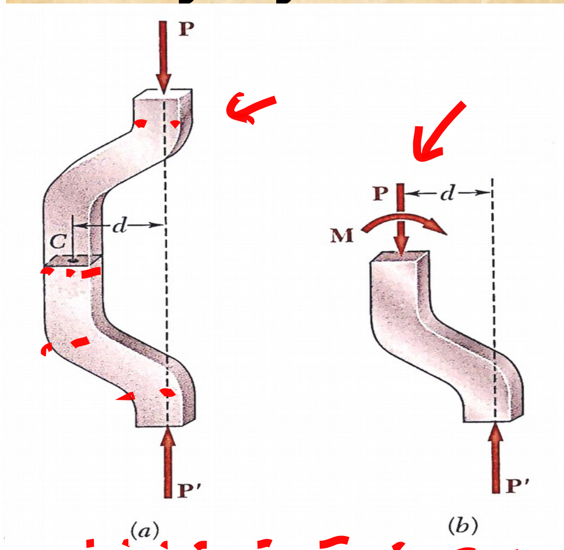

Fig. 1.14

Mini quiz 4

Question ~~2~~

Knowing that $P=160kN$, determine the average normal stress at the midsection of (a) rod **AB**, (b) rod **BC**.

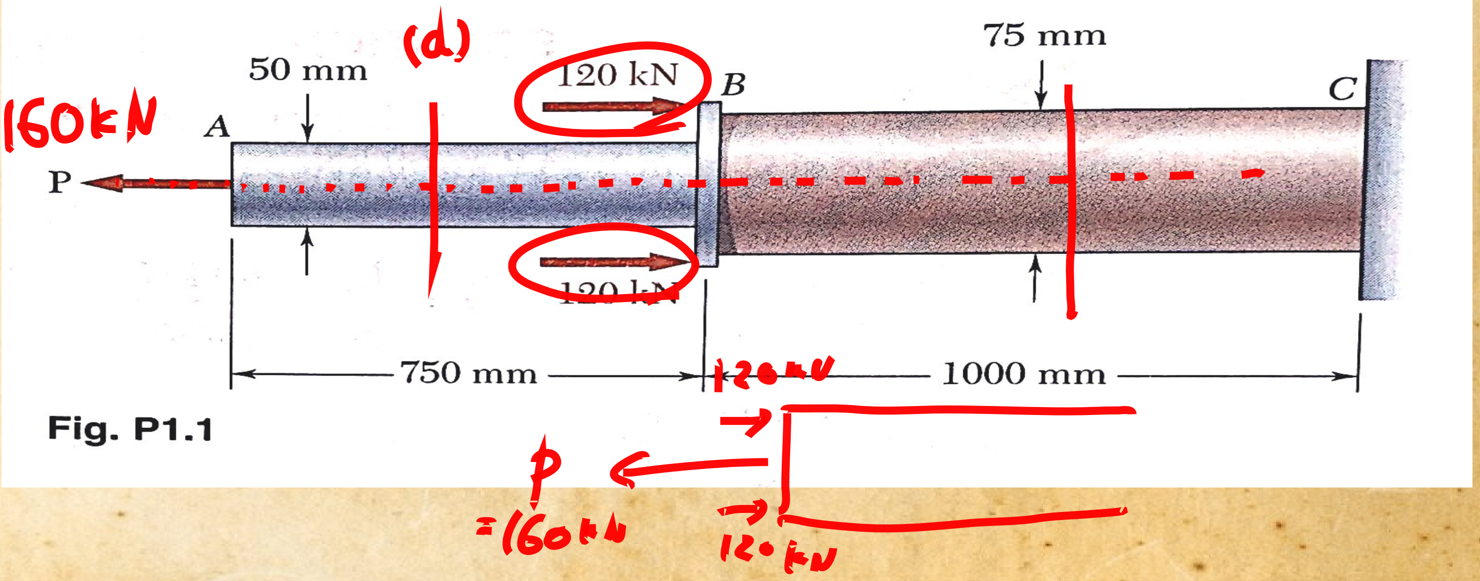

Fig. P1.1