Introduction into design engineering week 2

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Internal forces (member force)

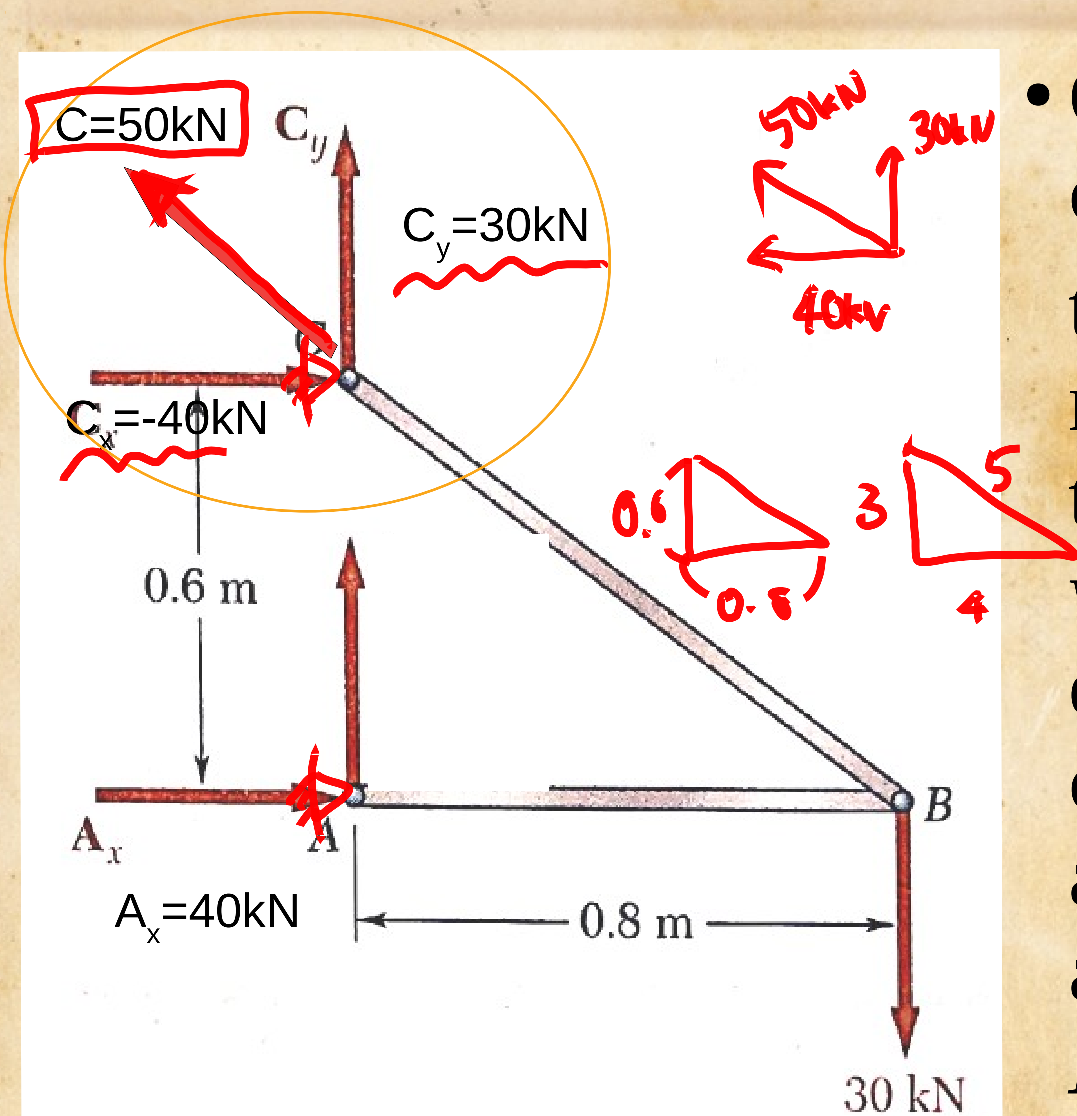


Fig. 1.2

- Observing that the components $\boldsymbol{C_x}$ and $\boldsymbol{C_y}$ of the reaction at ***C*** are, respectively, proportional to the *horizontal* and *vertical components* of the distance from ***B*** to ***C***, it is concluded that the reaction at ***C = 50 kN***, is directed along the axis of the rod ***BC***, and causes tension in that member.

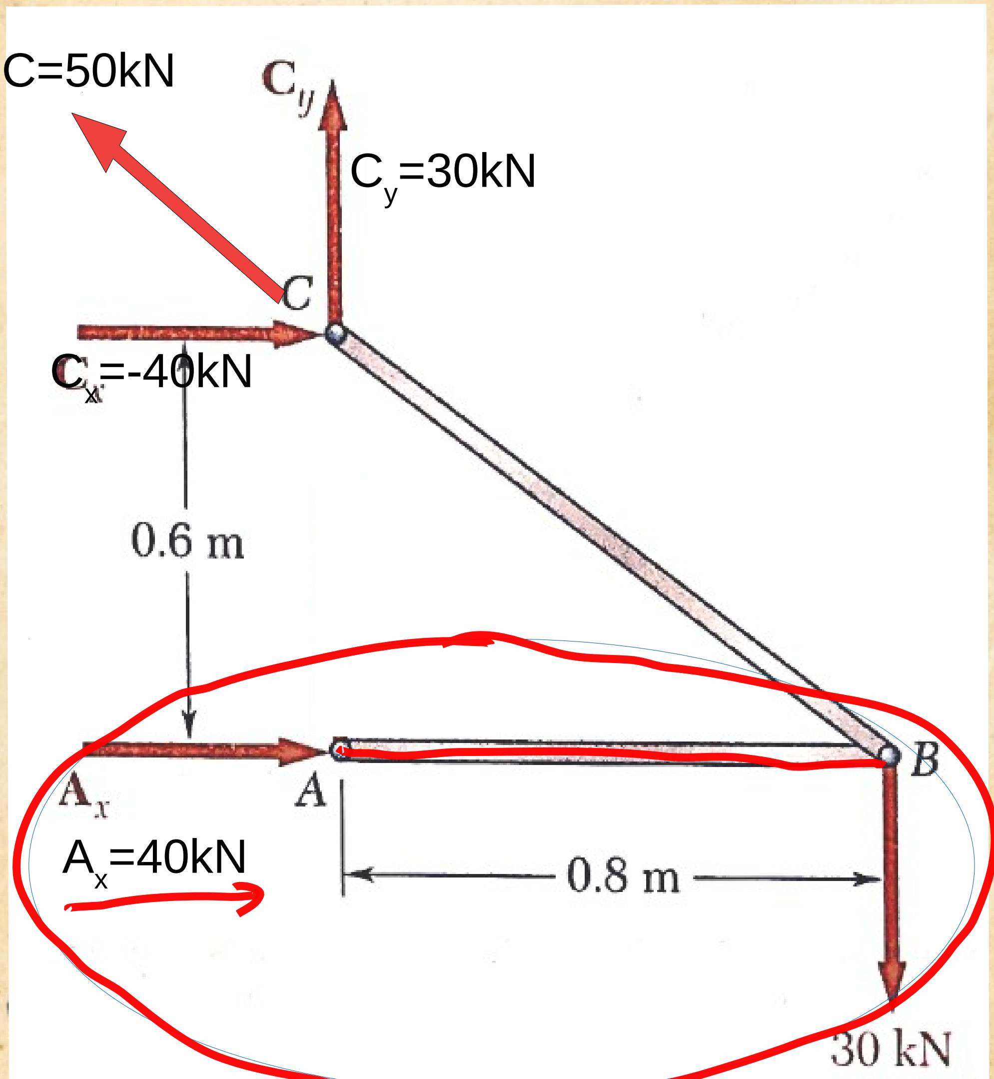


Fig. 1.2

Bondi

- At boom **AB**, reaction $\boldsymbol{A_x}$ is directed along the axis of this member and causes *compression* in that member.

directed

Another solution

- These results could have been anticipated by recognizing that ***AB*** and ***BC*** are *two-force* that means *members*, i.e., members that are subjected to forces at only two points, these points being ***A*** and ***B*** for member ***AB***, and ***B*** and ***C*** for member ***BC***.
- Indeed, for *a two-force member* the lines of action of the resultants of the forces acting at each of the two points are ***equal and opposite*** and pass through both points.

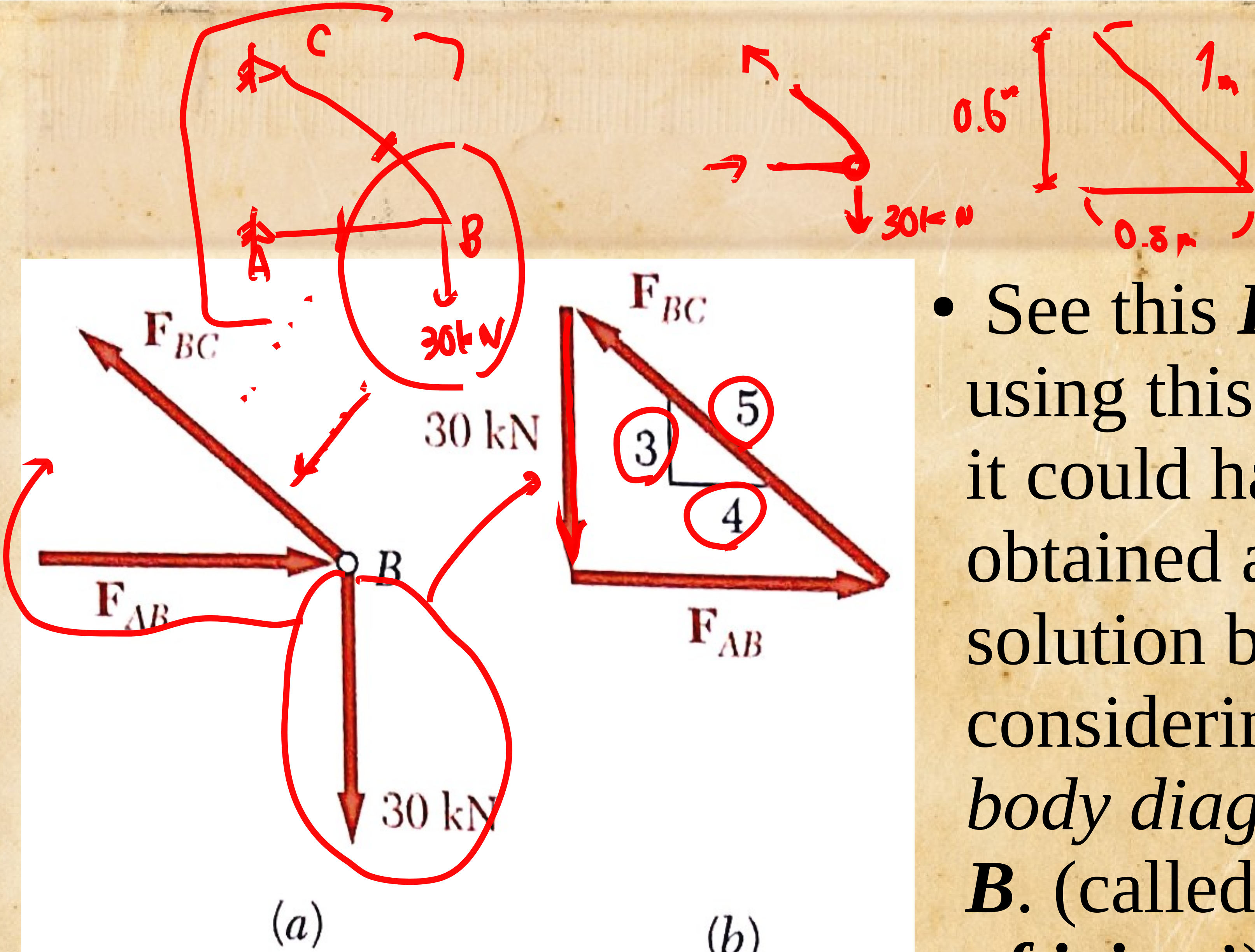


Fig. 1.4

- See this ***Fig.1.4***, using this property, it could have been obtained a simpler solution by considering the *free-body diagram* of pin ***B***. (called '***<u>method of joints</u>***').

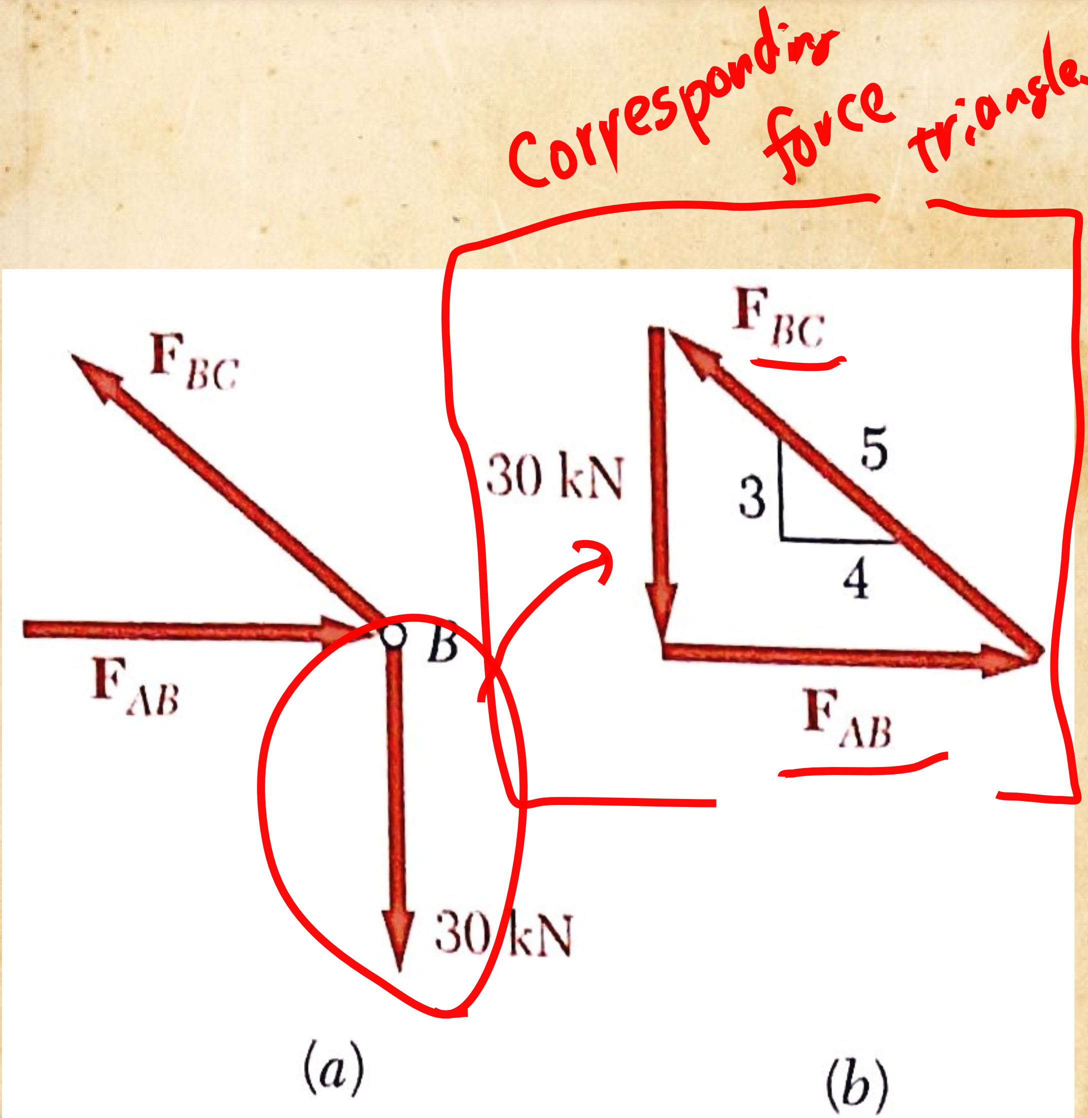
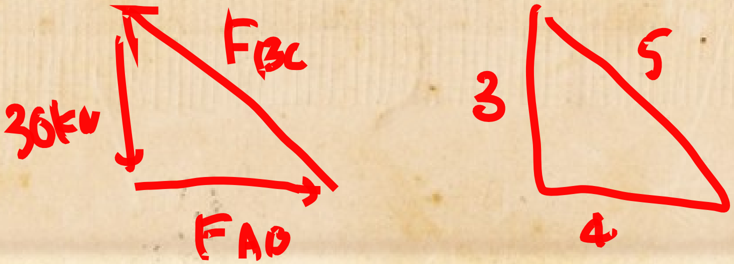


Fig. 1.4

- The forces on pin ***B*** and the forces $\boldsymbol{F_{AB}}$ and $\boldsymbol{F_{BC}}$ exerted, respectively, by members ***AB*** and ***BC***, and the ***30-kN*** load (*Fig.1.4a*).
- It can be expressed that pin ***B*** is in equilibrium by drawing the corresponding force triangle (*Fig. 1.4b*).



$\frac{1}{2}$ ←

- Since the force F_{BC} is directed along member ***BC***, its slope is the same as that of ***BC***, namely, ¾.

$\frac{3}{4}$ ←

fraction

- Therefore, it can be written the proportion;

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\text{ kN}}{3}$$

- From which it could be obtained

$$F_{AB} = 40\text{ kN} \qquad F_{BC} = 50\text{ kN}$$

- The forces F'_{AB} and F'_{BC} exerted by pin ***B***, respectively, on boom ***AB*** and rod ***BC*** are equal and opposite to F_{AB} and $\underline{\mathbf{F}_{BC}}$ (see in *Fig.1.5*).
- Knowing the forces at the ends of each of the members, it can now be determined the internal forces in these members.

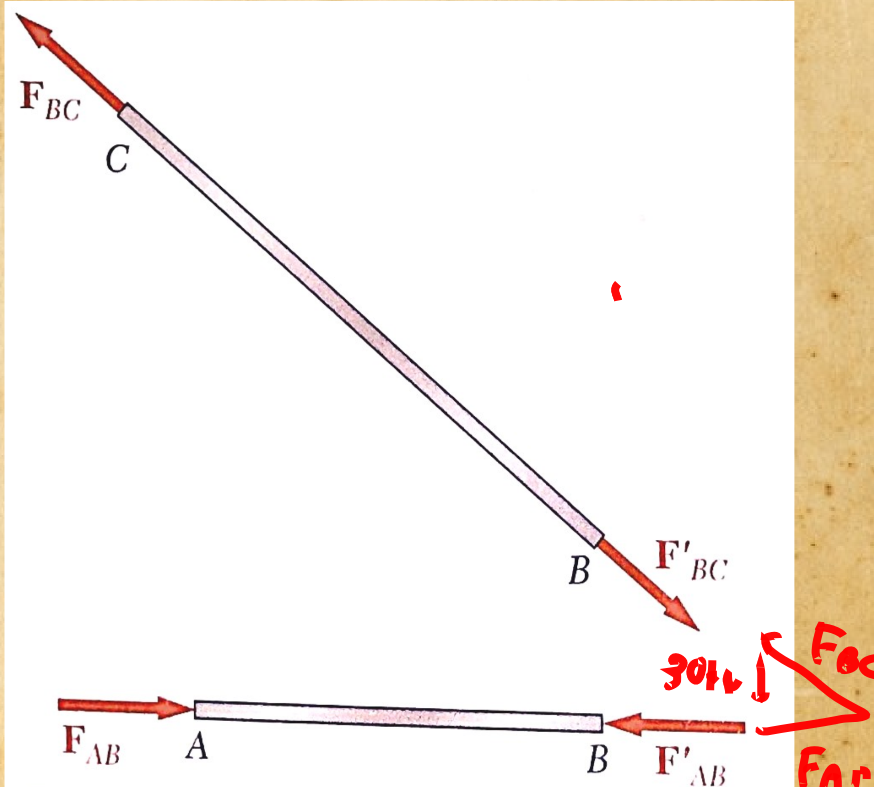


Fig. 1.5

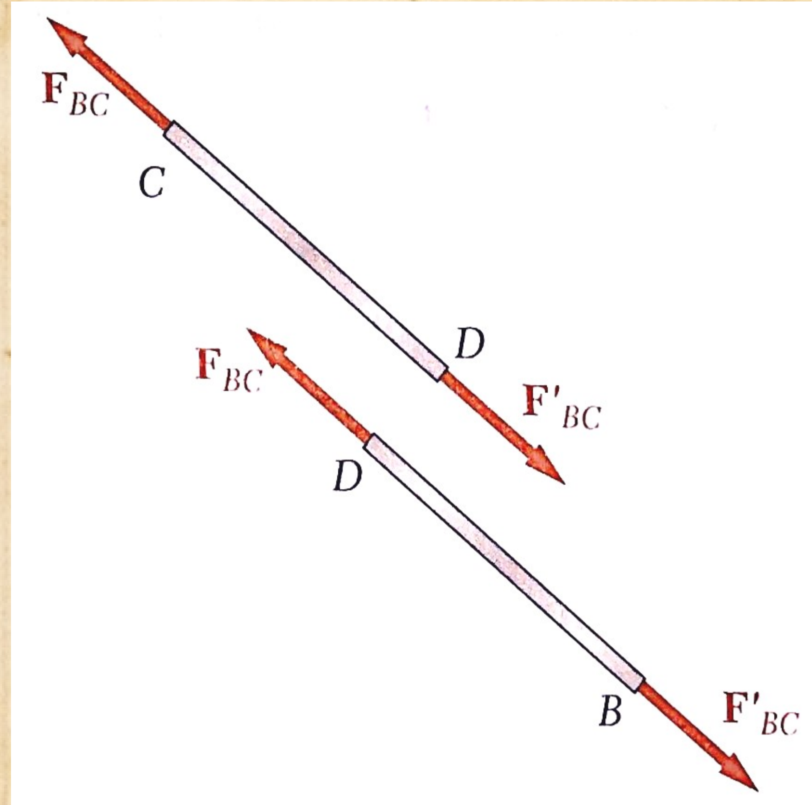


Fig. 1.6

- Passing a section at some arbitrary point ***D*** of rod ***BC,*** it is obtained two portions ***BD*** and ***CD*** (*Fig.1.6*).
- Since *50-kN forces* must be applied at ***D*** to both portions of the rod to keep them in equilibrium, it is concluded that an internal force of *50-kN* is produced in rod **BC** when a *30-kN* load is applied at ***B***.

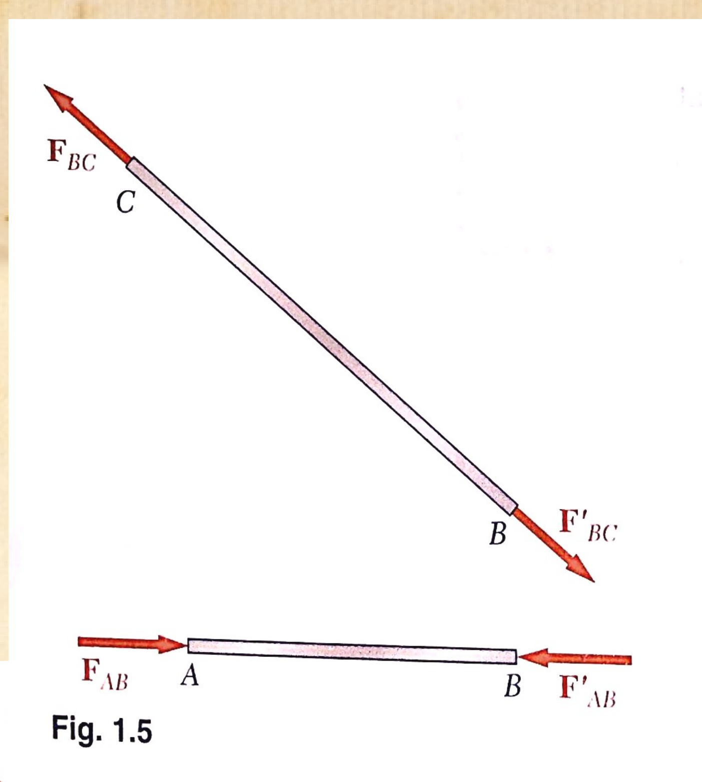


Fig. 1.5

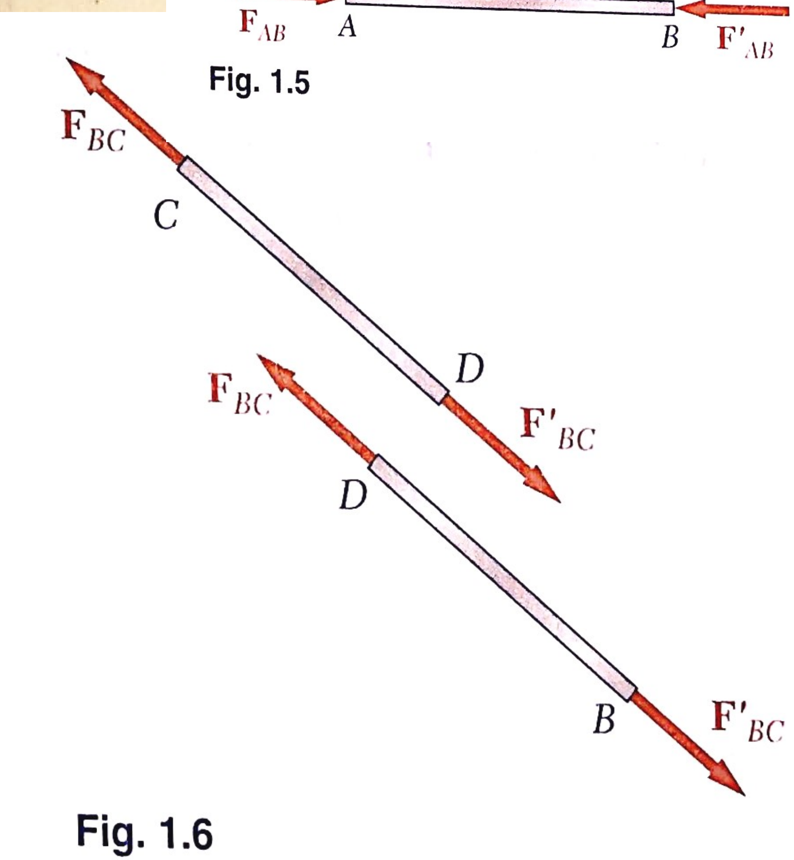
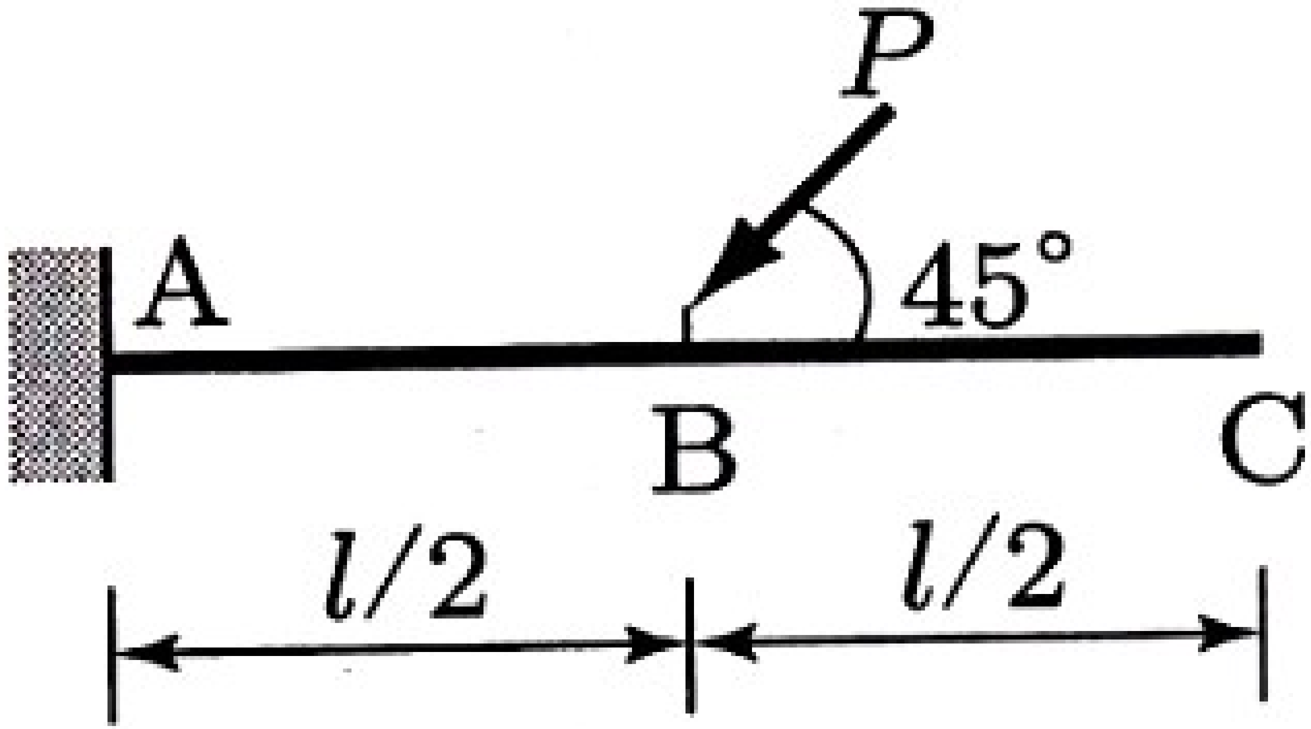


Fig. 1.6

- It would be further checked from the directions of the forces $\boldsymbol{F}_{BC}$ and $\boldsymbol{F'}_{BC}$ in *Fig. 1.6* that the rod is in tension.
- A similar procedure would enable us to determine that the internal force in boom ***AB*** is *40 kN* and that the boom is in compression.

Example question



Stresses in the members of a structure

- While the results obtained from last week's class (such that we found reactions and internal forces) represent a first and necessary step in the analysis of the given structure.
- However, these values can not tell us whether the given load can be safely supported.

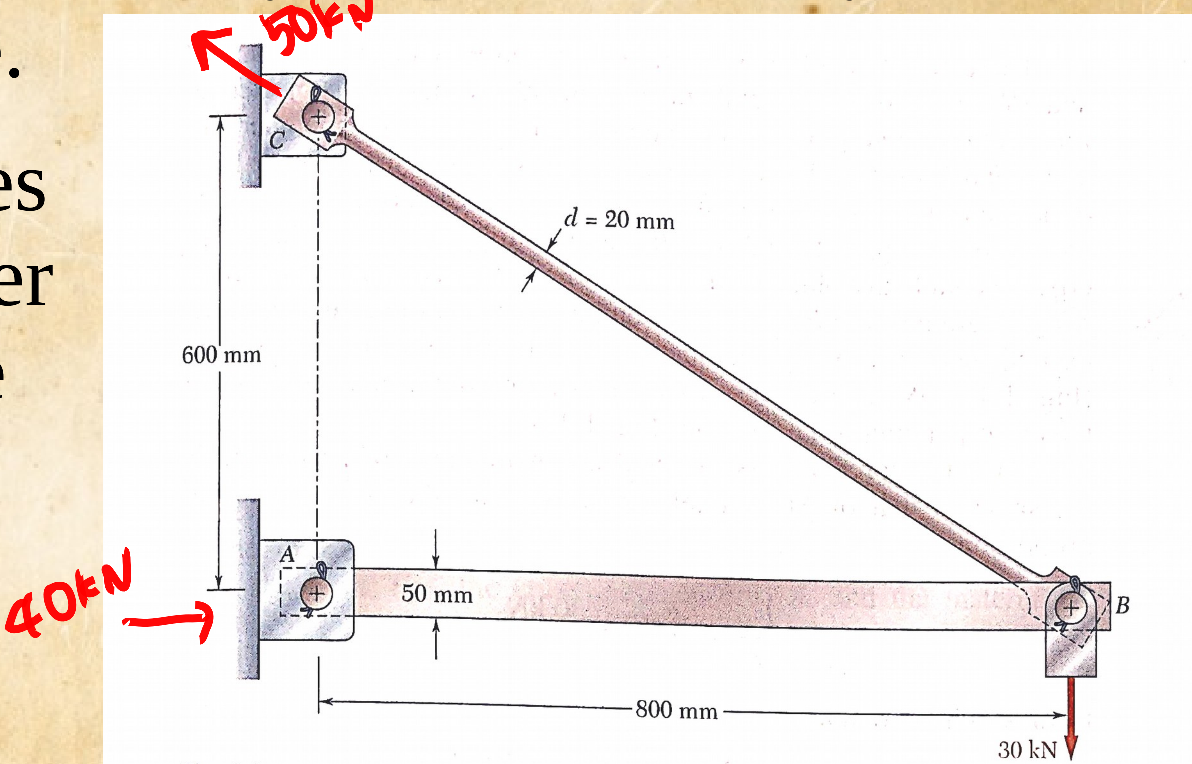


Fig. 1.1

- For example, whether rod ***BC*** will break or not under this loading depends not only upon the value found for the internal force $\boldsymbol{F_{BC}}$,but also upon *the cross-sectional area* of the rod and the material of which the rod is made.

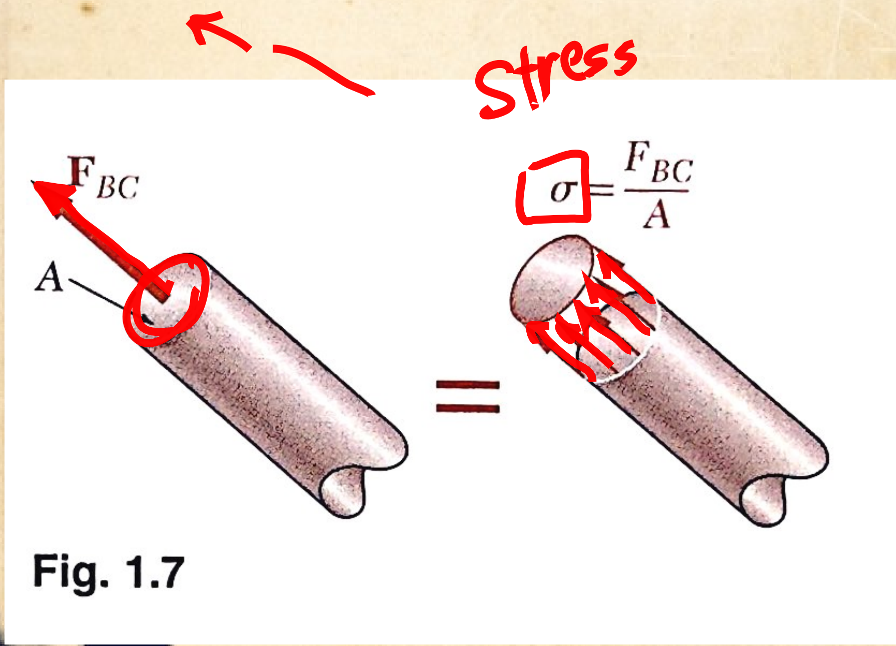


Fig. 1.7

- In deed, the internal force $\boldsymbol{F_{BC}}$ actually represents the resultant of elementary forces distributed over the entire area ***A*** of the cross section (*Fig 1.7*) and the average intensity of these distributed forces is equal to the force per unit area, $\boldsymbol{F_{BC}/A}$, in the section.

- Whether or not the rod will break under the given loading clearly depends upon the ability of the material to withstand the corresponding value $\boldsymbol{F_{BC}/A}$ of the intensity of the distributed internal forces.

- It thus depends upon the force $\boldsymbol{F_{BC}}$, the cross-sectional area $\boldsymbol{A}$, and the material of the rod.

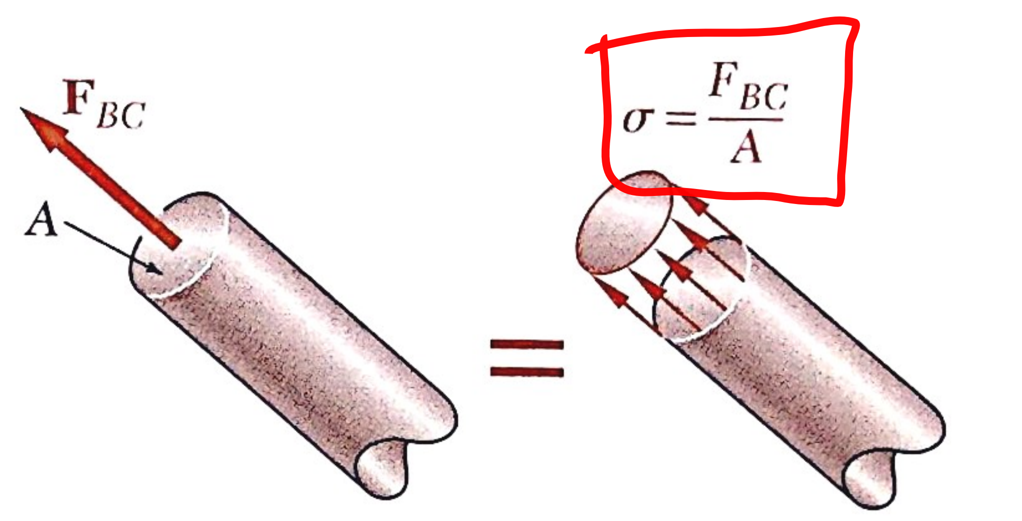


Fig. 1.7

- The force per unit area, or intensity of the forces distributed over a given section, is called the ***stress*** on that section and is denoted by the Greek letter ***σ (sigma)***.
- The stress in a member of cross-sectional area ***A*** subjected to an axial load ***P*** (*Fig. 1.8*) is therefore obtained by dividing the magnitude ***P*** of the load by the area ***A***:

$$\boxed{\sigma = \frac{P}{A}} \qquad (1.5)$$

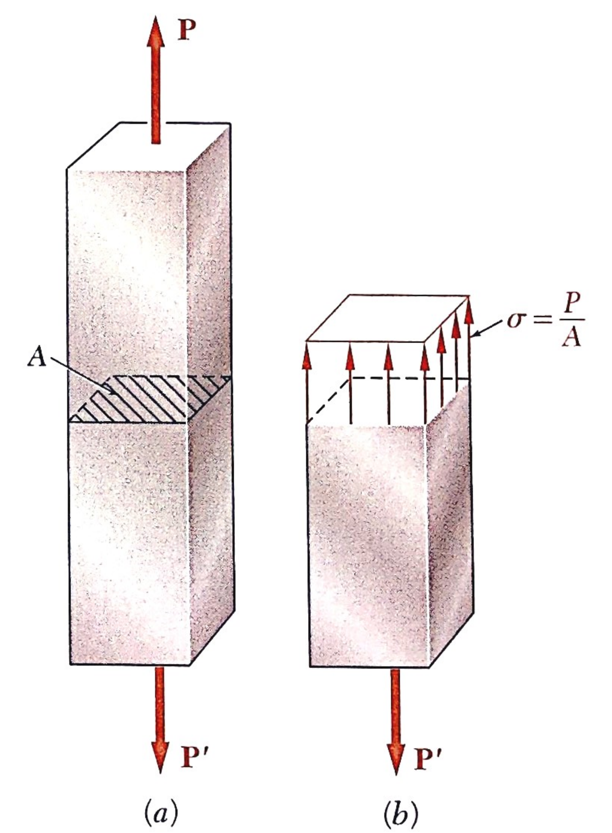


Fig. 1.8

x^2

- A *positive sign* will be used to indicate a ***tensile stress*** (member in tension) and a *negative sign* to indicate a ***compressive stress*** (member in compression).
- Since *SI metric* units are used in this discussion, with ***P*** expressed in newtons(**N**) and ***A*** in square meters (m^2), the stress ***σ*** will be expressed in N/m^2.
- This unit is called a *pascal* (Pa).

10^2 10^3 x^7

- However, on e finds that the *pascal* is an exceedingly small quantity and that, in practice, multiples of this unit must be used, namely, the *kilo-pascal* **(kPa)**, the *mega-pascal* (**MPa**), and the *giga-pascal* (**GPa**), we have

$$1\ \text{kPa} = 10^3\ \text{Pa} = 10^3\ \text{N/m}^2$$

$$1\ \text{MPa} = 10^6\ \text{Pa} = 10^6\ \text{N/m}^2$$

$$1\ \text{GPa} = 10^9\ \text{Pa} = 10^9\ \text{N/m}^2$$

1,350,000,000 Pa

1.35 GPa

ANALYSIS and DESIGN

Analysis

- Considering again the structure of *Fig.1.1*, let us assume that rod ***BC*** is made of a steel with a maximum allowable stress $\boldsymbol{\sigma_{all}=165MPa}$**.**
- Can rod ***BC*** safely support the load to which it will be subjected?

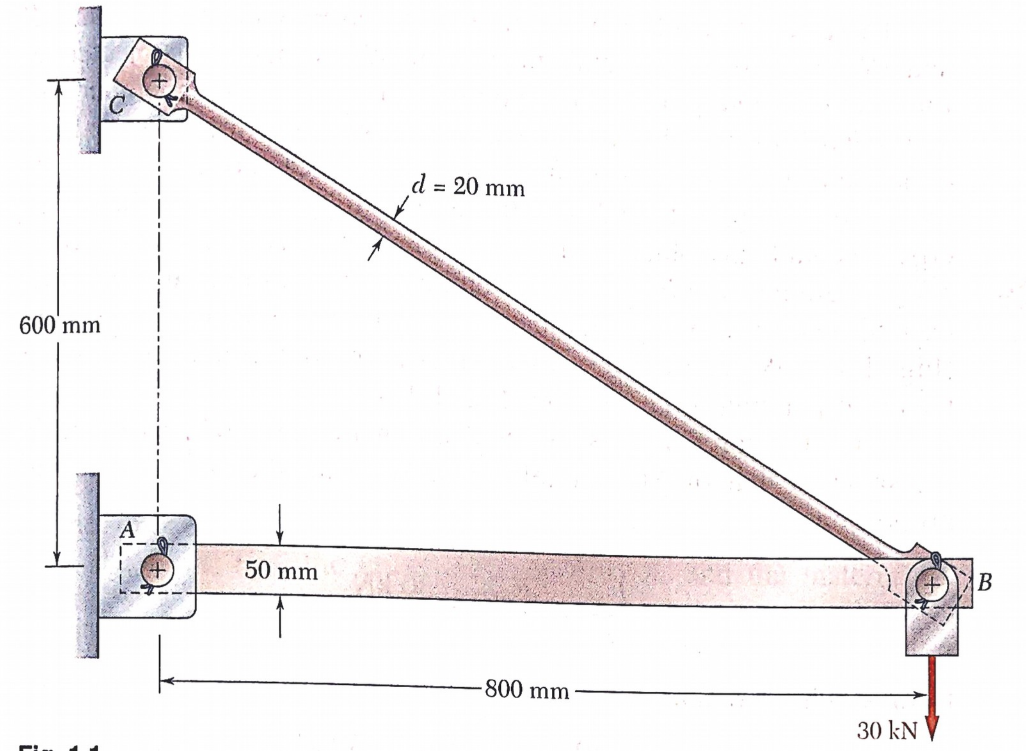


Fig. 1.1

- Find ***P*** in N.

$$P = F_{BC} = +50\text{ kN} = +50 \times 10^3\text{ N}$$

- Determine ***A*** in mm2.

$$A = \pi r^2 = \pi\left(\frac{20\text{ mm}}{2}\right)^2 = \pi(10 \times 10^{-3}\text{ m})^2 = 314 \times 10^{-6}\text{ m}^2$$

- Finally find ***σ*** in Mpa (N/mm2).

$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3\text{ N}}{314 \times 10^{-6}\text{ m}^2} = +159 \times 10^6\text{ Pa} = +159\text{ MPa}$$

- Since the value obtained for ***σ (=159 MPa)*** is smaller than the value ***σ_{all} (=165 Mpa)*** of the allowable stress in the steel used, it is concluded that rod ***BC*** can safely support the load to which it will be subjected.
- To be complete, this analysis of the given structure should also include the determination of the *compressive stress* in boom ***AB***, as well as am investigation of the produced in the pins and their bearings.

Design

- The engineer's role is not limited to the analysis of existing structures and machines subjected to given loading conditions.
- Of even greater importance to the engineer is <u>*the design of new structures and machines*</u>, that is, the selection of appropriate components to perform a given task.

Example of Design using Fig.1.1

- Now let's assume that <u>*aluminum*</u> with an allowable stress $\boldsymbol{\sigma_{all}=100MPa}$ is to be used.
- Since the force in rod ***BC*** will still be

$$\boldsymbol{P=F_{BC}=50kN}$$

under the given loading applied,

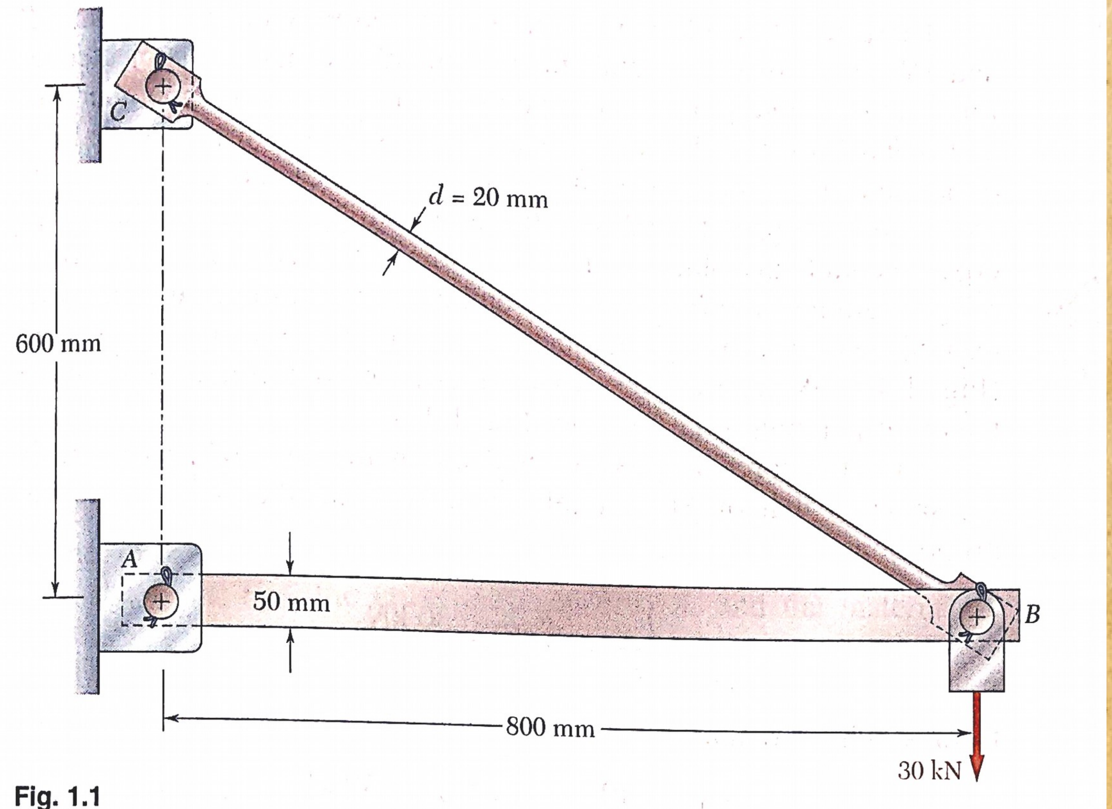


Fig. 1.1

- Since the value obtained for ***σ (=159 MPa)*** is bigger than the value ***σ_{all} (=100 Mpa)*** of the allowable stress in the aluminum used, it must be re-designed the ***Area of rod BC*** by the following equation;

$$\sigma_{\text{all}} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{\text{all}}} = \frac{50 \times 10^3 \text{ N}}{100 \times 10^6 \text{ Pa}} = 500 \times 10^{-6} \text{ m}^2$$

- Since ***A=πr²***,

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{500 \times 10^{-6}\ \mathrm{m}^2}{\pi}} = 12.62 \times 10^{-3}\ \mathrm{m} = 12.62\ \mathrm{mm}$$

$$d = 2r = 25.2\ \mathrm{mm}$$

- It is concluded that an *aluminum rod* ***26 mm or more in diameter*** will be adequate.

Question 1

Two solid cylindrical rods ***AB*** and ***BC*** are welded together at ***B*** and loaded as shown.

Determine the magnitude of the force ***P*** for which the tensile stress in rod ***AB*** is twice the magnitude of the compressive stress in rod ***BC***.

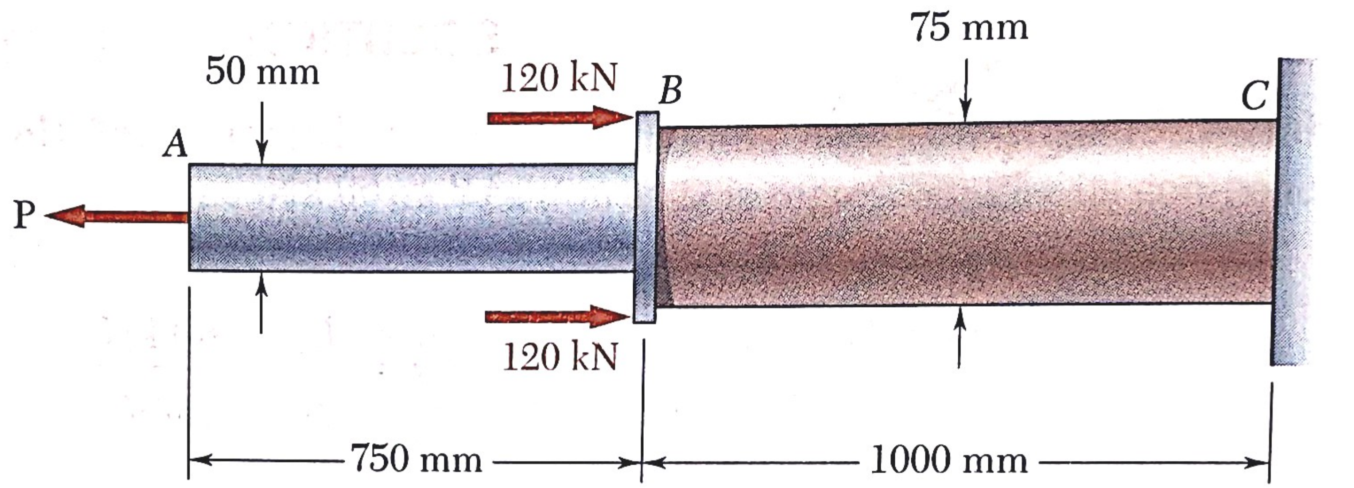


Fig. P1.1