Introduction into design engineering week 1

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Introduction

- ***Objectives***

  The main objectives of this class should be to develop in the engineering students the

  1: Ability to analyze a given problem in simple and logical manner

  2: Apply to its solution a few fundamental and well-understood principles.

Week1: introduction of this week

- The basic methods of statics

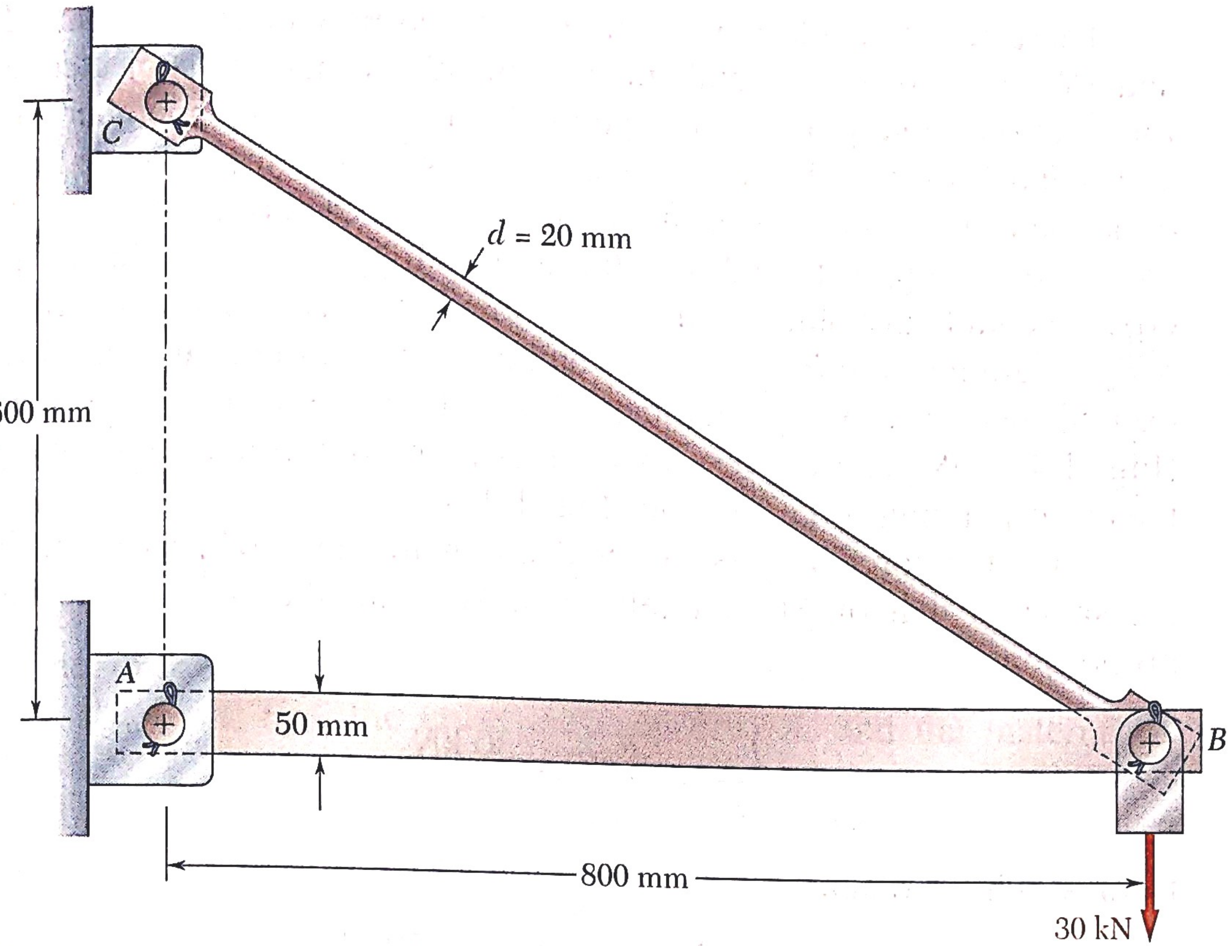


Fig. 1.1

The basic methods of statics

- Consider the structure shown in Fig.1.1, which was designed to support a ***30-kN*** load.

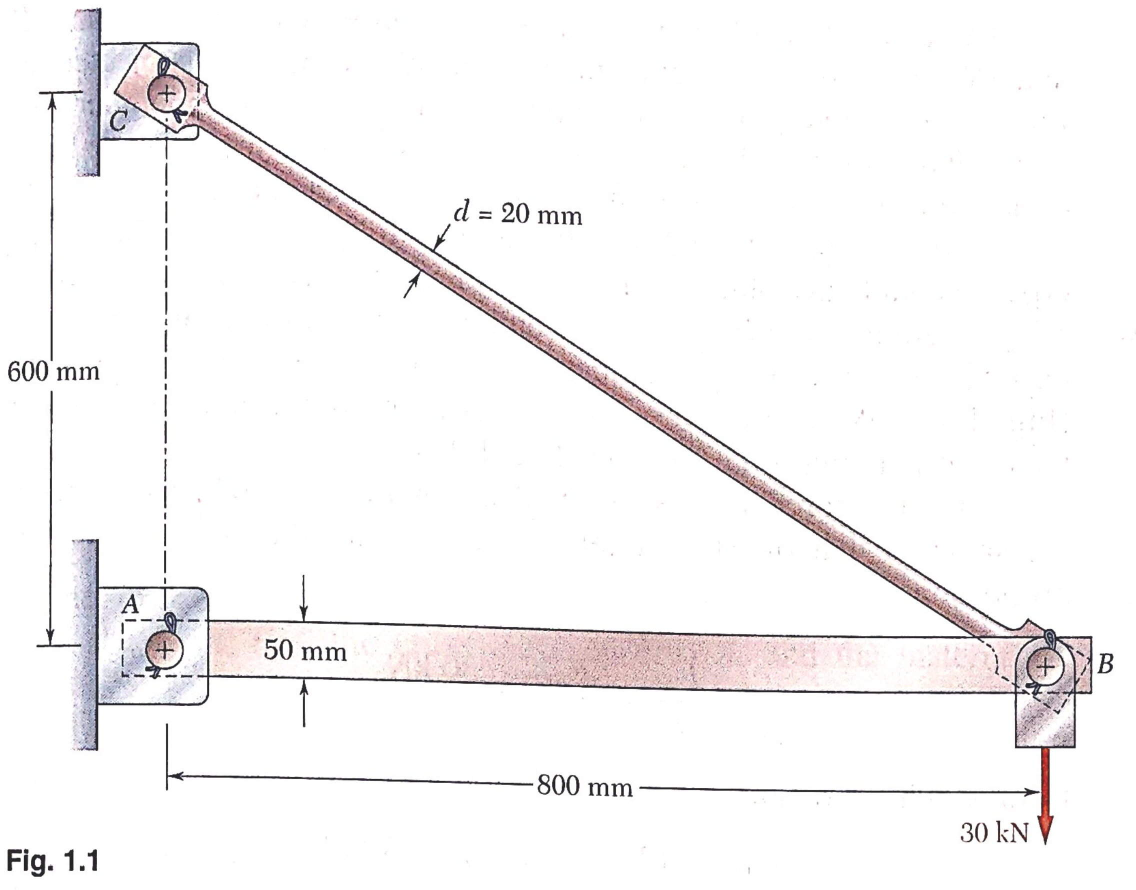
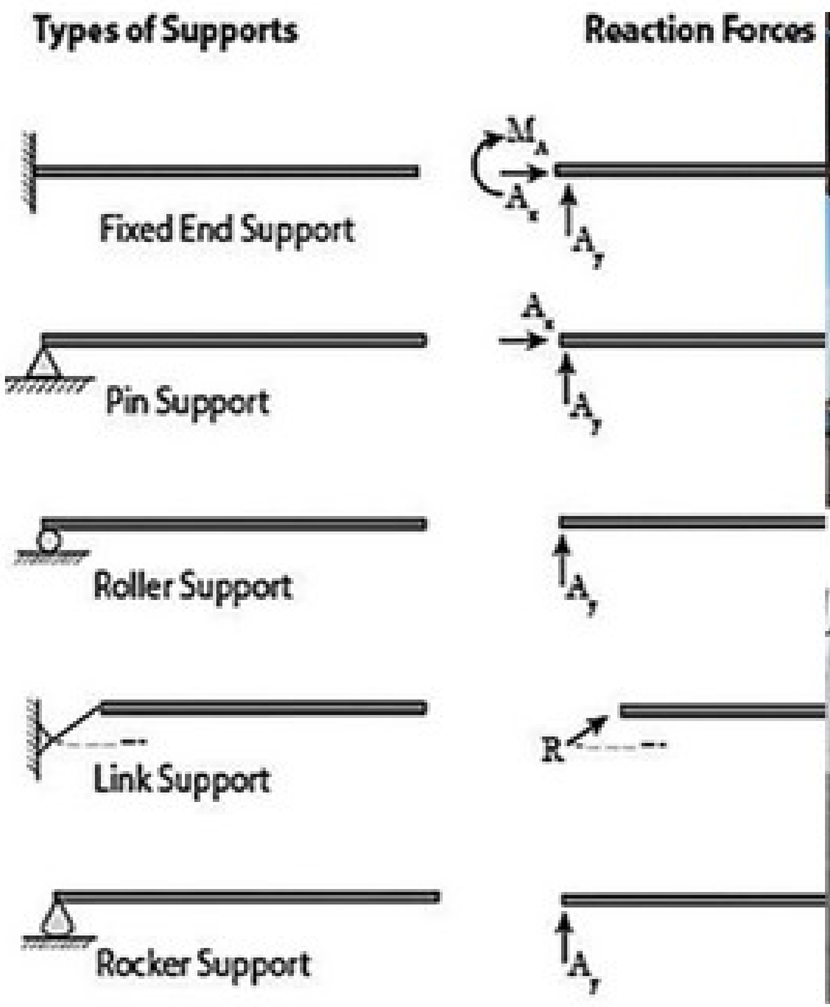


Fig. 1.1

- It consists of a boom ***AB*** with a *30 × 50 mm* rectangular cross section and of rod ***BC*** with a 20-mm-diameter circular cross section.
- The *boom* and the *rod* are connected by a pin at ***B*** and are supported by *pins* and *brackets* at ***A*** and ***C*** respectively.

Support conditions



Step 1: free-body diagram

- ***Free-body diagram***

  Throughout the text *free-body diagrams* are used to determine external or internal forces.

  The use of *'picture equations'* will also help the students understand the superposition of loadings and the resulting stresses and deformations.

Draw a Free-body diagram

- Free-body diagram of the structure by detaching it from its supports at ***A*** and ***C***, and showing the reactions that these supports exert on the structure (*Fig.1.2*).

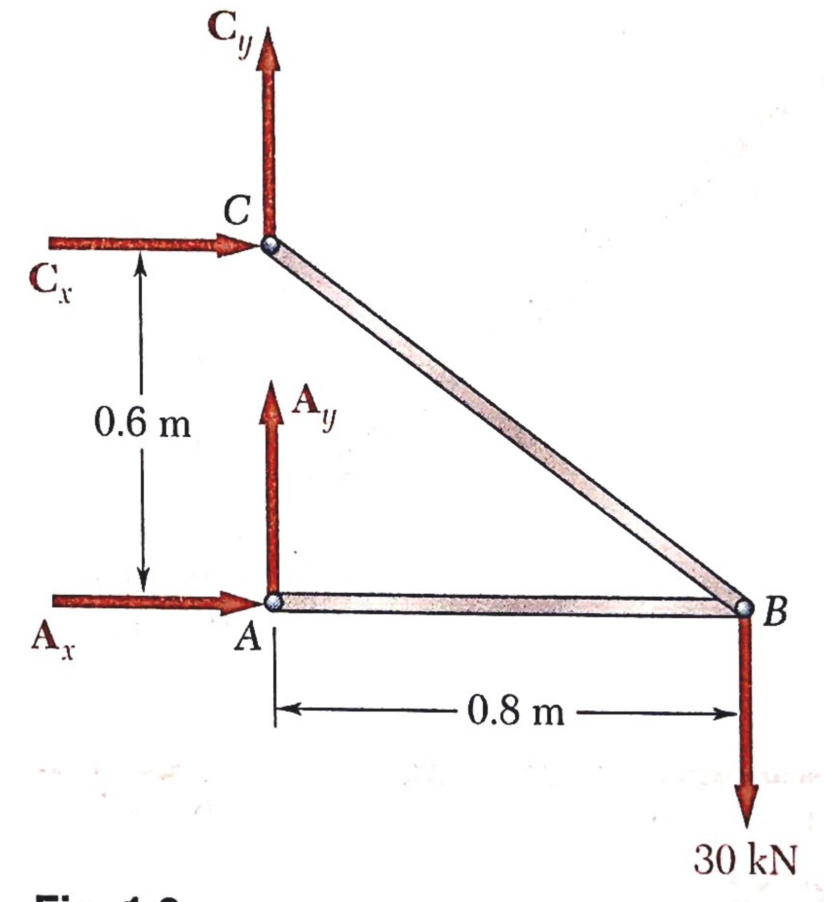


Fig. 1.2

- Note that the sketch of the structure has been simplified by omitting all unnecessary details.
- Many of you may have recognized at this point that ***AB*** and ***BC*** are ***two-force members***.

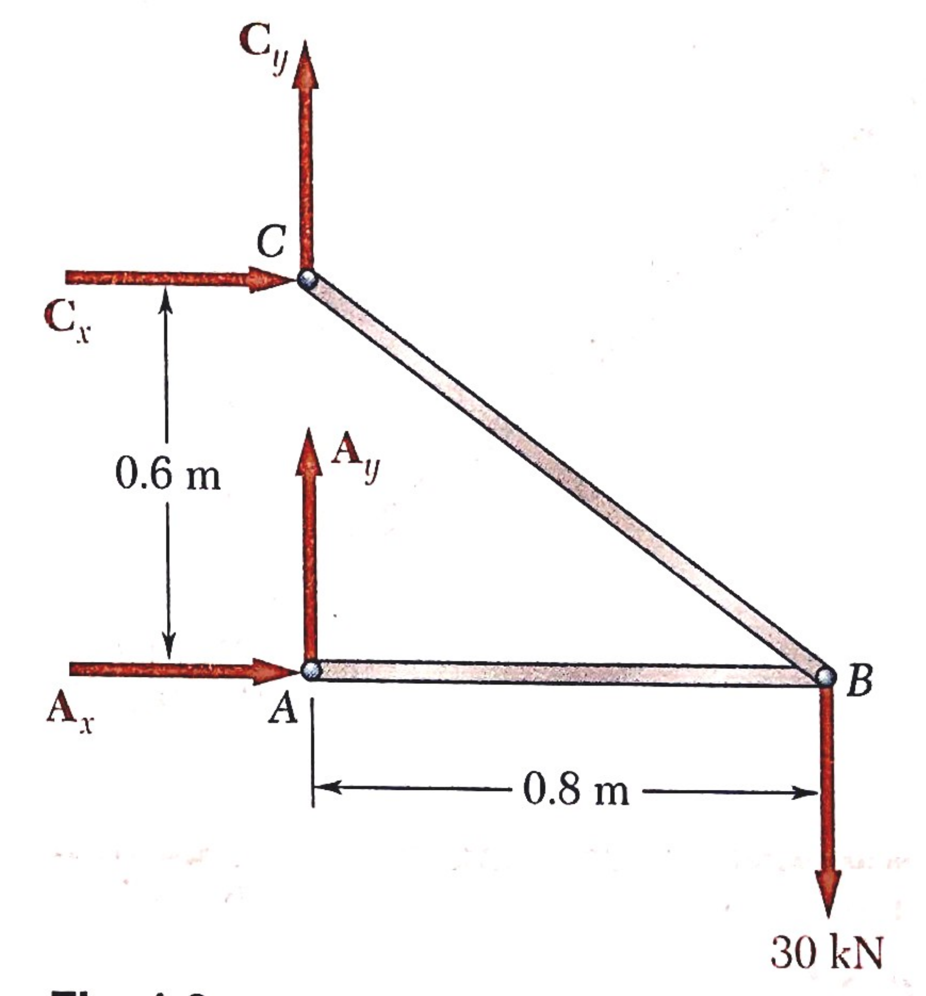


Fig. 1.2

- For those of you who have not, we will pursue our analysis, ignoring that fact and assuming that the directions of the reactions at ***A*** and ***C*** are unknown.
- Each of these reactions, therefore, will be represented by two components, $\boldsymbol{A_x}$ and $\boldsymbol{A_y}$ at ***A***, and $\boldsymbol{C_x}$ and $\boldsymbol{C_y}$ at ***C***.

Step 2: Solve equations to get reactions

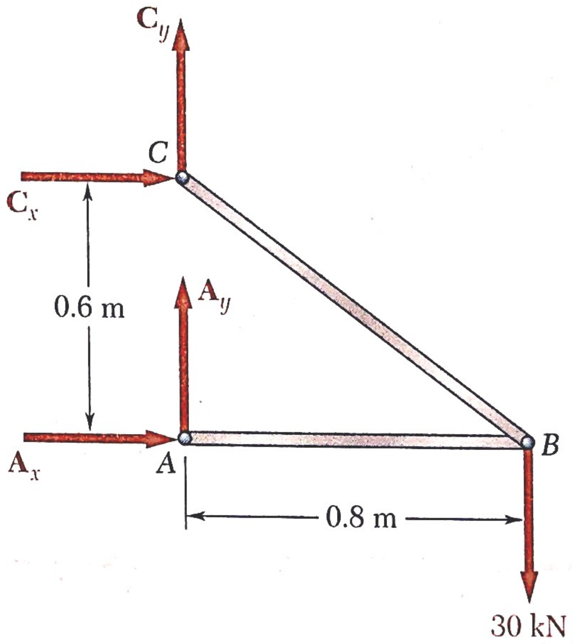


Fig. 1.2

- ***Moment***
- ***Horizontal force***
- ***Vertical force***

$$+\circlearrowleft \Sigma M_C = 0: \quad A_x(0.6\text{ m}) - (30\text{ kN})(0.8\text{ m}) = 0$$

$$A_x = +40\text{ kN} \tag{1.1}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad A_x + C_x = 0$$

$$C_x = -A_x \qquad C_x = -40\text{ kN} \tag{1.2}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + C_y - 30\text{ kN} = 0$$

$$A_y + C_y = +30\text{ kN} \tag{1.3}$$

- A_y and C_y are still unknown

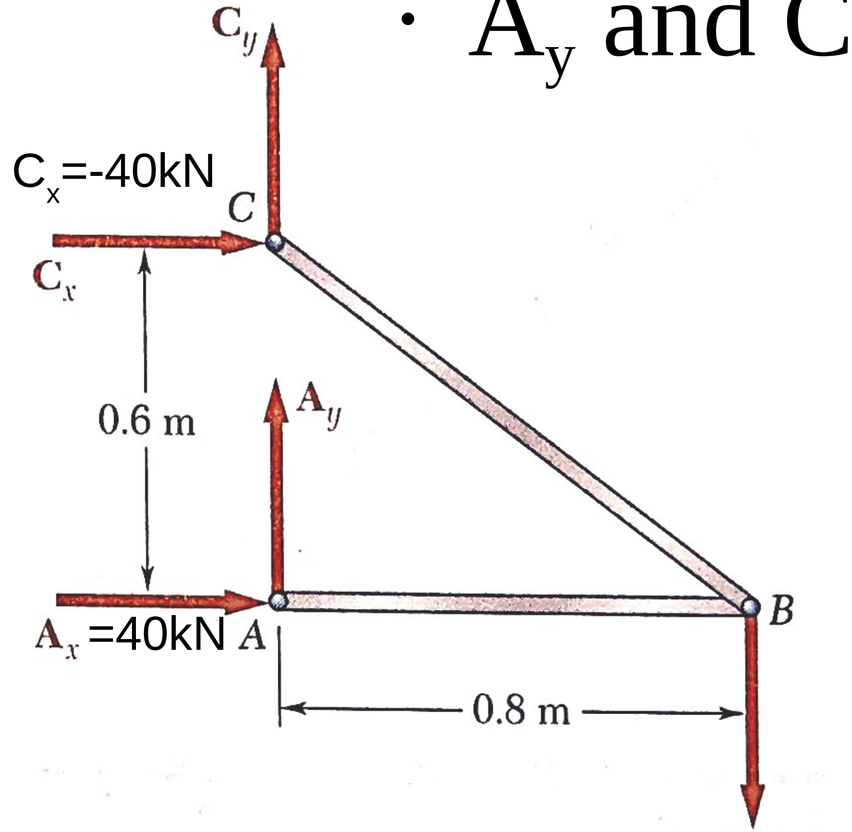


Fig. 1.2

- A_y and C_y are still unknown

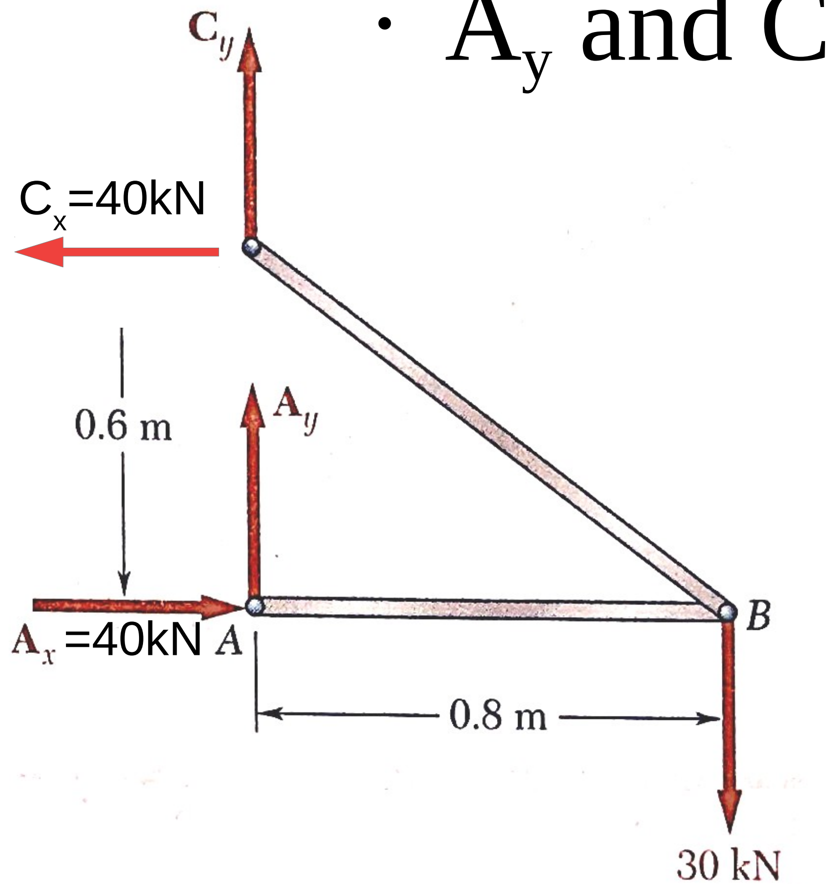


Fig. 1.2

Step 3: free-body diagram 2

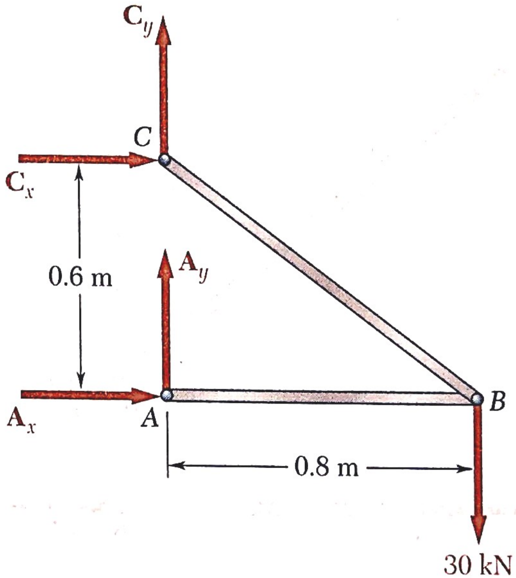


Fig. 1.2

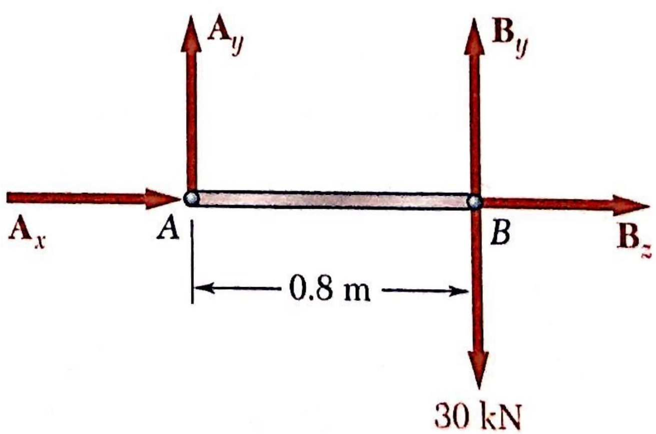


Fig. 1.3

- There are only ***3 equations*** to solve reactions for one free-body diagram.
- Thus, it should be now dismembered the structure.
- Considering the free-body diagram of the ***boom AB*** (see in Fig.1.3), then we can use three equations again and write this:

$$+\circlearrowleft \Sigma M_B = 0:$$

$$-A_y(0.8 \text{ m}) = 0$$

$$A_y = 0 \qquad (1.4)$$

Summary of all reactions.

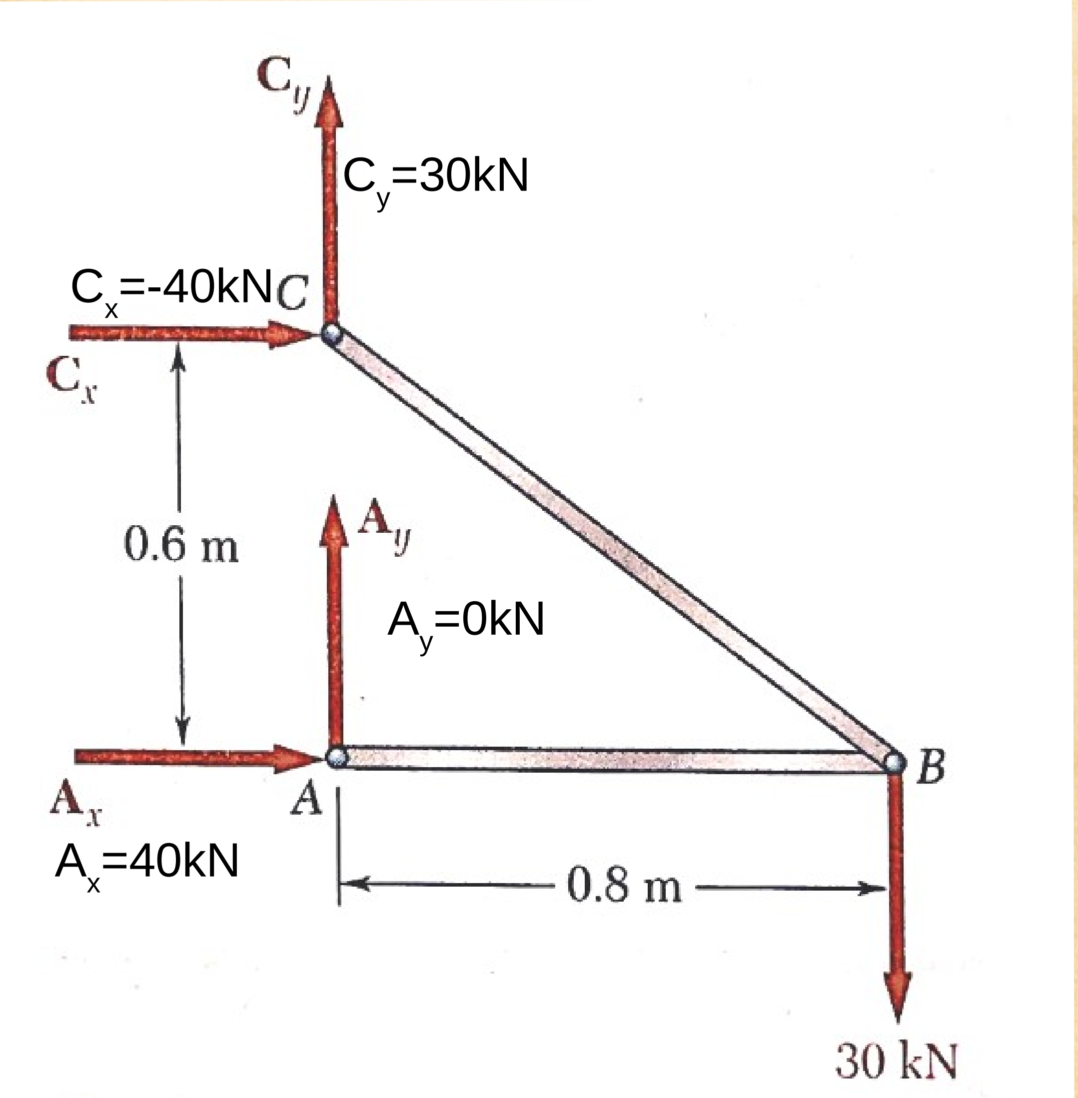


Fig. 1.2

- Back to *equation 1.3*

$$A_y + C_y = +30 \text{ kN}$$

- Now $\mathbf{A_y}$ is ***0***, thus $\boldsymbol{C_y}$ is ***30 kN***.
- All reactions are found on this structure!

Internal forces (member force)

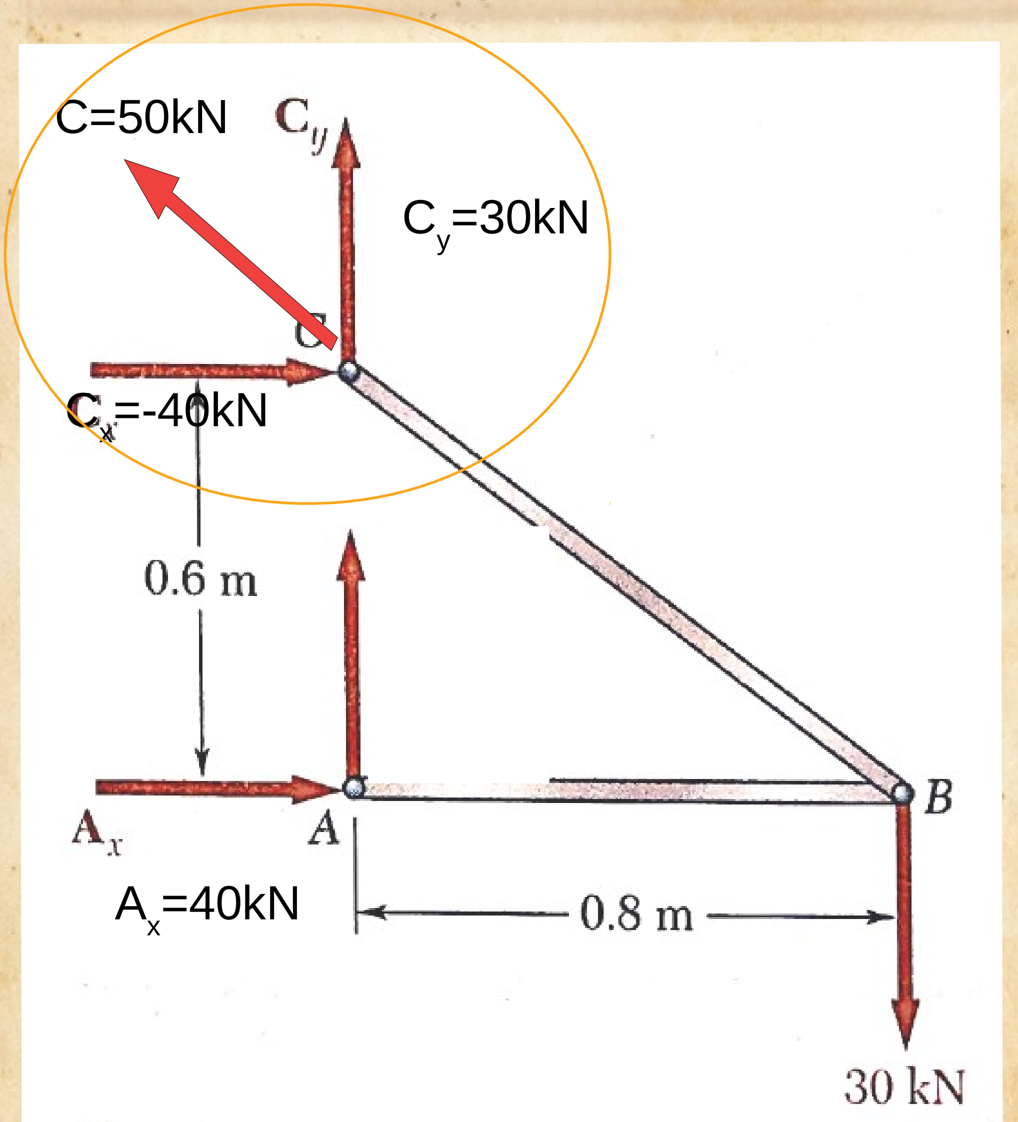


Fig. 1.2

- Observing that the components $\boldsymbol{C_x}$ and $\boldsymbol{C_y}$ of the reaction at **C** are, respectively, proportional to the *horizontal* and *vertical components* of the distance from ***B*** to ***C***, it is concluded that the reaction at ***C = 50 kN***, is directed along the axis of the rod ***BC***, and causes tension in that member.

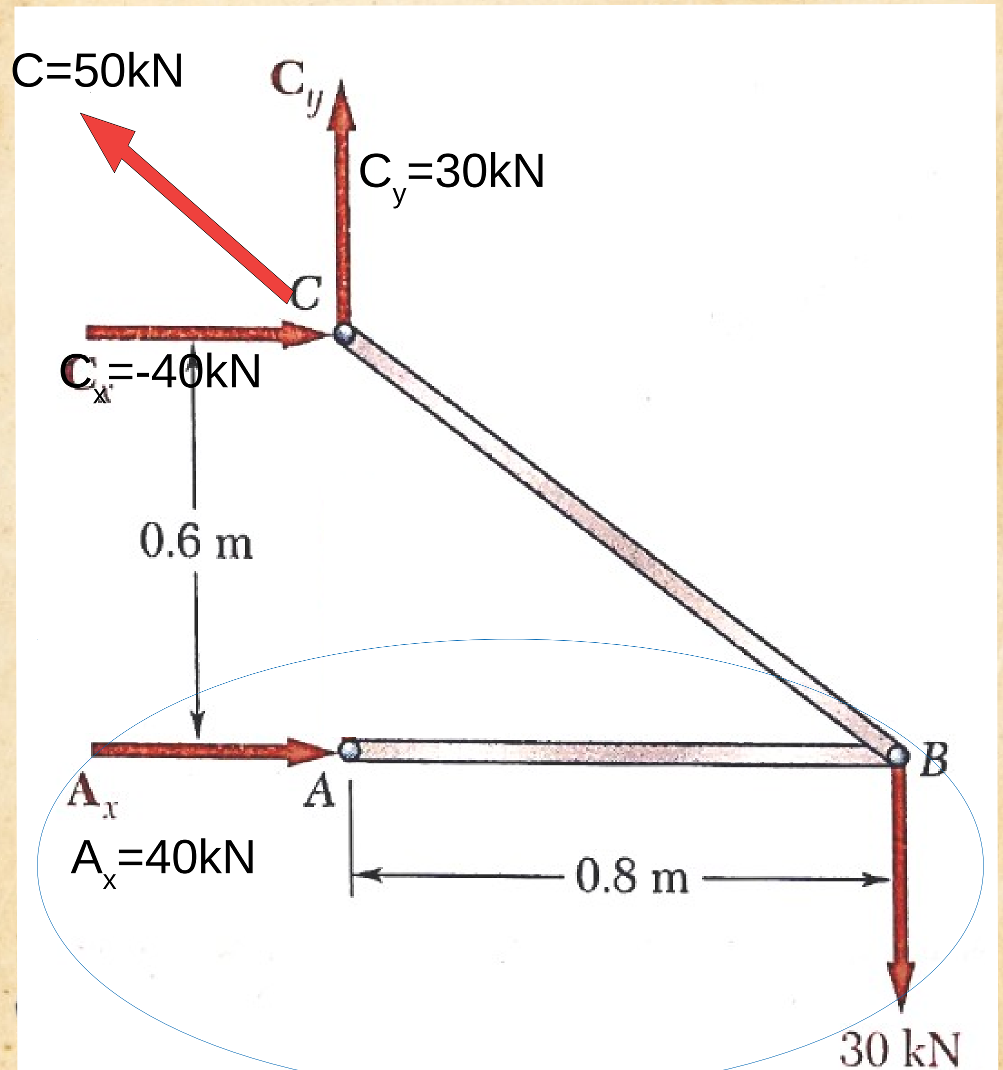


Fig. 1.2

- At boom **AB**, reaction A_x is directed along the axis of this member and causes *compression* in that member.

Another solution

- These results could have been anticipated by recognizing that ***AB*** and ***BC*** are *two-force members*, i.e., members that are subjected to forces at only two points, these points being ***A*** and ***B*** for member ***AB***, and ***B*** and ***C*** for member ***BC***.
- Indeed, for *a two-force member* the lines of action of the resultants of the forces acting at each of the two points are ***equal and opposite*** and pass through both points.

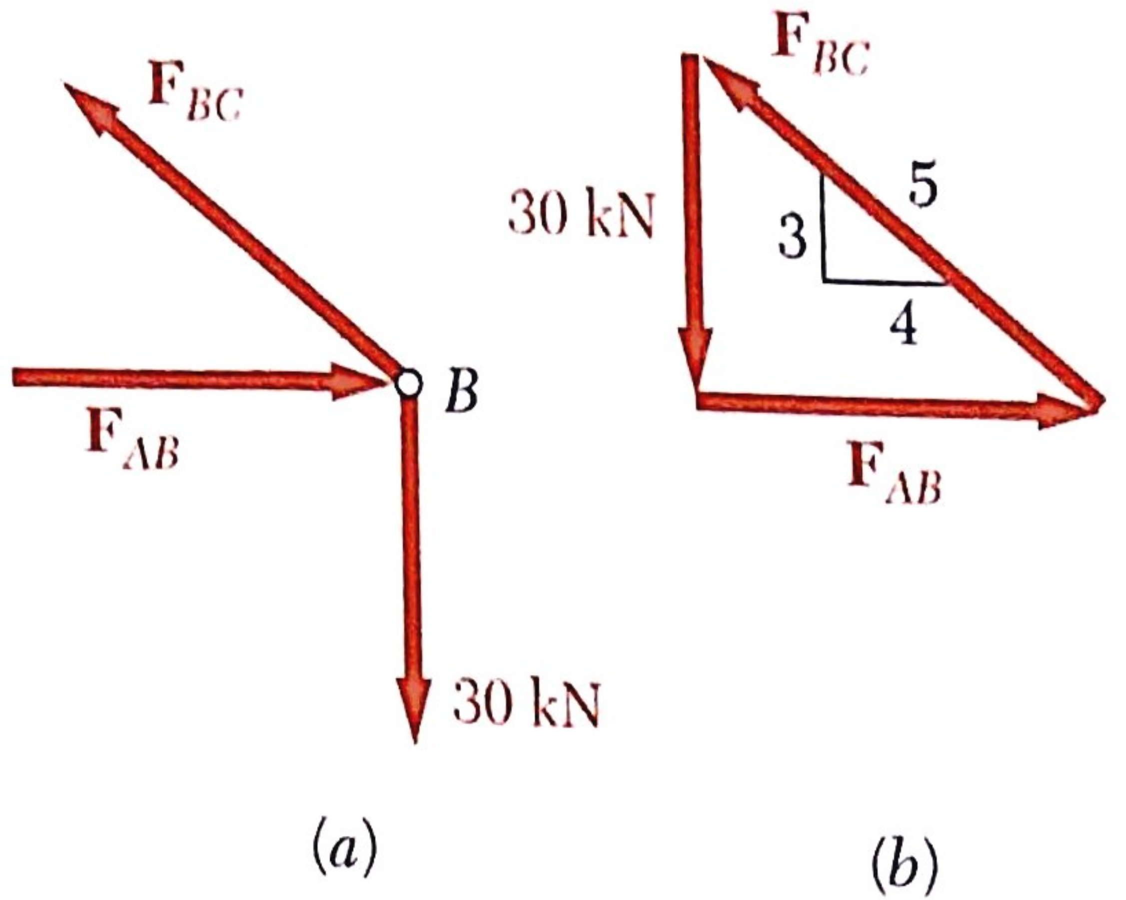


Fig. 1.4

- See this ***Fig.1.4***, using this property, it could have been obtained a simpler solution by considering the *free-body diagram* of pin ***B***. (called '***method of joints***').

- The forces on pin ***B*** and the forces $\boldsymbol{F_{AB}}$ and $\boldsymbol{F_{BC}}$ exerted, respectively, by members ***AB*** and ***BC***, and the ***30-kN*** load (*Fig.1.4a*).
- It can be expressed that pin ***B*** is in equilibrium by drawing the corresponding force triangle (*Fig. 1.4b*).

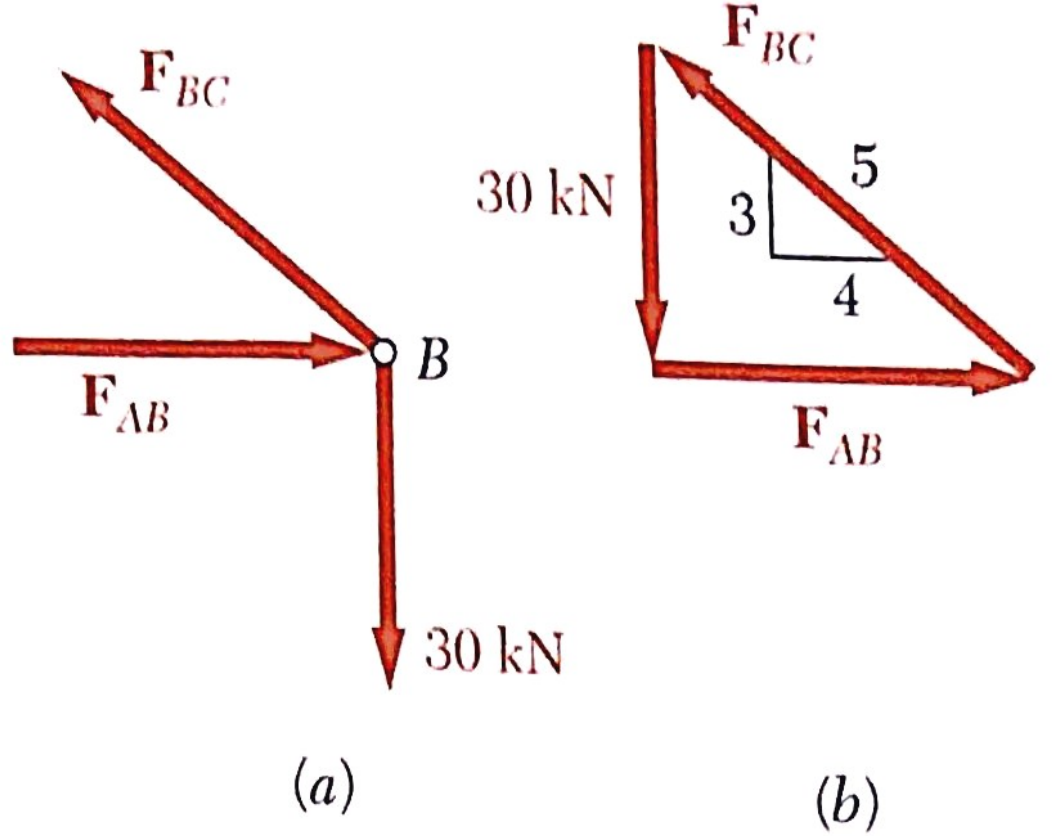


Fig. 1.4

- Since the force $\boldsymbol{F_{BC}}$ is directed along member ***BC***, its slope is the same as that of ***BC***, namely, ***¾***.
- Therefore, it can be written the proportion;

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\ \text{kN}}{3}$$

- From which it could be obtained

$$F_{AB} = 40\ \text{kN} \qquad F_{BC} = 50\ \text{kN}$$

- The forces $\boldsymbol{F'_{AB}}$ and $\boldsymbol{F'_{BC}}$ exerted by pin ***B***, respectively, on boom ***AB*** and rod ***BC*** are equal and opposite to $\boldsymbol{F_{AB}}$ and $\mathbf{F_{BC}}$ (see in *Fig.1.5*).
- Knowing the forces at the ends of each of the members, it can now be determined the internal forces in these members.

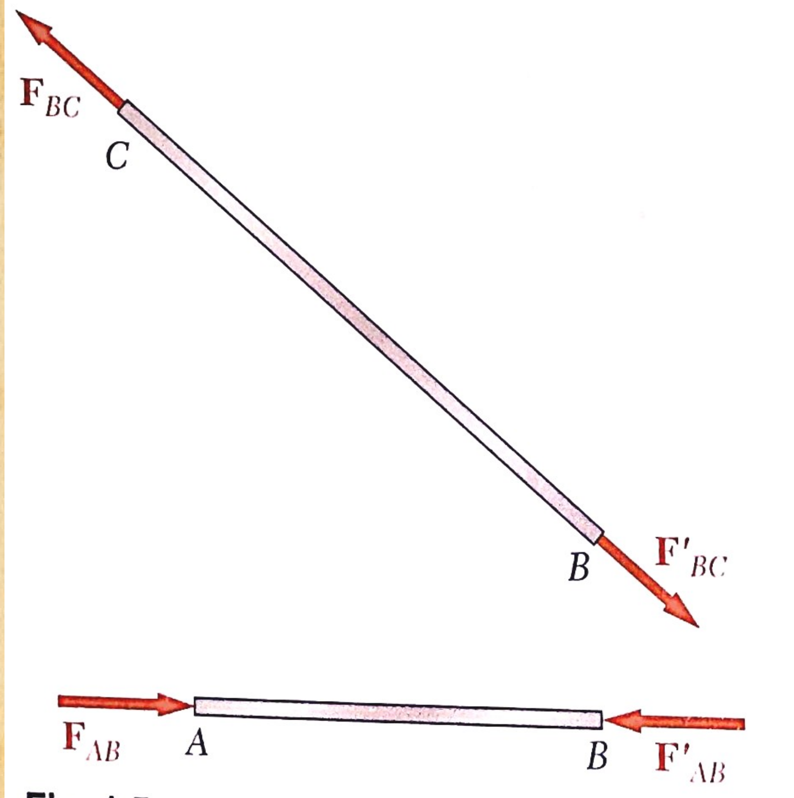


Fig. 1.5

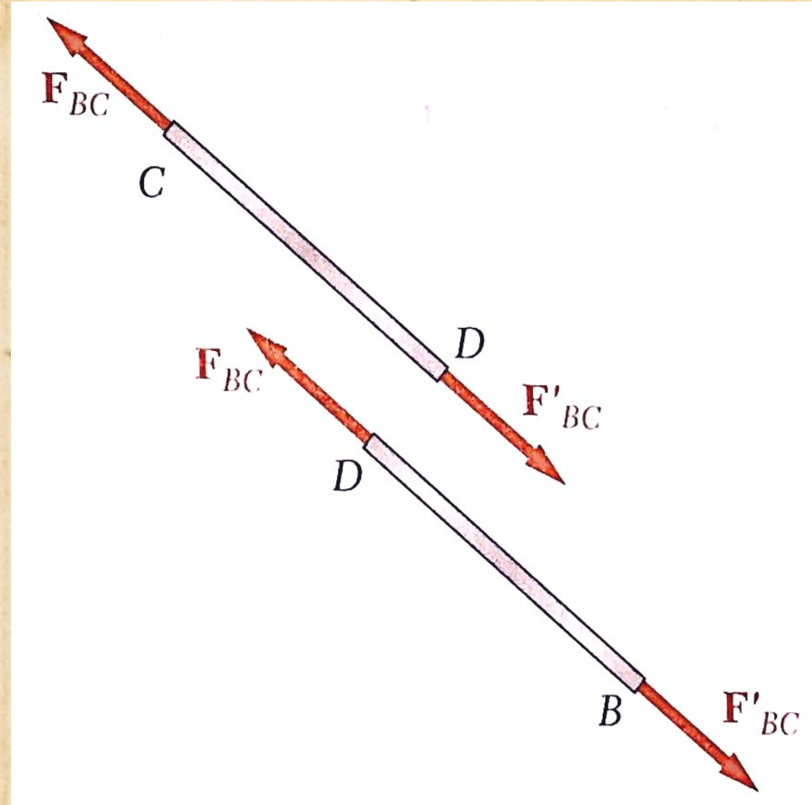


Fig. 1.6

- Passing a section at some arbitrary point ***D*** of rod ***BC*****,** it is obtained two portions ***BD*** and ***CD*** (*Fig.1.6*).
- Since *50-kN forces* must be applied at ***D*** to both portions of the rod to keep them in equilibrium, it is concluded that an internal force of *50-kN* is produced in rod **BC** when a *30-kN* load is applied at ***B***.

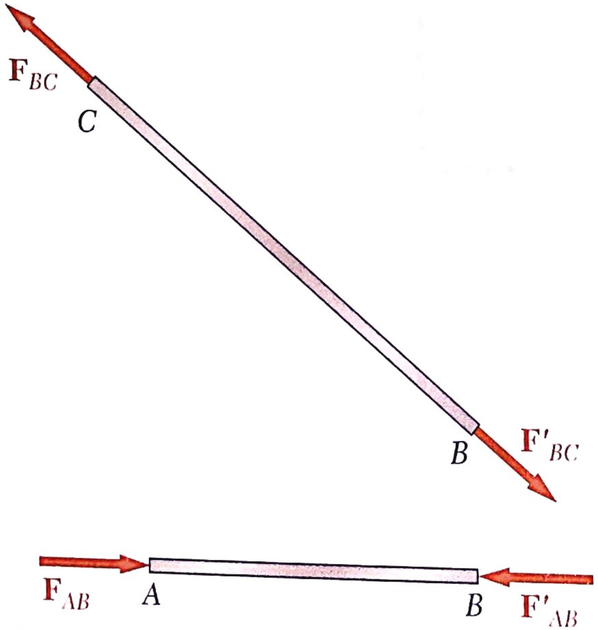


Fig. 1.5

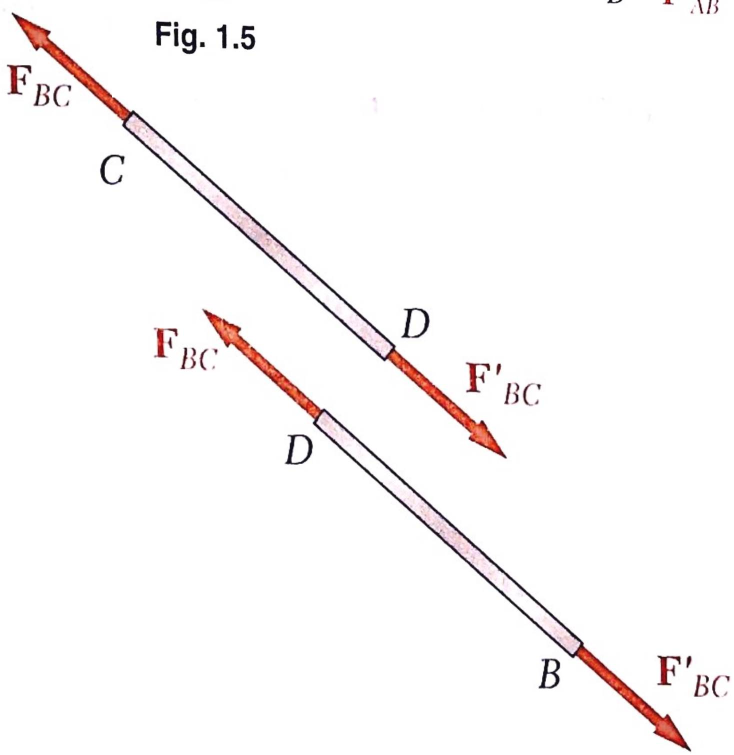


Fig. 1.6

- It would be further checked from the directions of the forces $\boldsymbol{F_{BC}}$ and $\boldsymbol{F'_{BC}}$ in *Fig. 1.6* that the rod is in tension.
- A similar procedure would enable us to determine that the internal force in boom ***AB*** is *40 kN* and that the boom is in compression.